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# Inventory and Corporate Risk Management\*

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## ABSTRACT

We consider a dynamic model of investment in which a firm can hold inventory to mitigate the price risk of an input commodity. Our model predicts that inventory allows to hedge against net worth risk by smoothing investment in capital, irrespective of the level of current net worth. Savings enhance the operational hedge offered by inventory, because they better conserve net worth when the commodity price is low. These predictions are confirmed in a sample of U.S. manufacturing corporations. We find that the empirical sensitivity of inventory investment to price changes is positive for any level of the firm's net worth. While savings and inventory are both positively related to financing constraints and cash flow risk, investment is more sensitive to inventory.

**JEL Classification:** G31, G32.

**Keywords:** Corporate Risk Management, Inventory, Cash Holdings.

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# Introduction

Inventory management is perhaps the most traditional and common operational hedge used by corporations. However, over the last fifty years U.S. corporations have drastically reduced inventory, owing to high storage costs on the one side, and improvements in supply chain efficiency, outsourcing, and reliance on multiple suppliers on the other.<sup>1</sup> Not surprisingly, the very technological and regulatory innovations that have allowed such a reduction in inventory holdings have exposed firms to new risks. These risks and the historical increase in domestic and global competition have recently rekindled managers' interest in inventory vis-à-vis other risk management tools, like cash holdings. Chen, Frank, and Wu (2005) argue that firms prefer to hold inventory to manage the supply chain risks that just-in-time production or reliance on multiple suppliers cannot eliminate. One such risk is commodity price uncertainty.

Relative to other hedges (e.g., derivatives), inventory has been little explored in corporate finance. In this paper, we study the use of inventory for net worth risk management, and therefore focus on a risk management motive for storing raw materials, whereby firms hold inventory to mitigate the effects of input price shocks. By analyzing the role of inventory as an operational hedge, we make a theoretical contribution to the risk management literature, which has so far mainly focused on either noncontingent tools (cash holdings) or collateralized contingent instruments (e.g., derivative contracts and lines of credit). In this respect, inventory is a contingent tool that does not need to be collateralized—the implications of which have not been fully analyzed.

We capture the contribution of inventory to risk management in a model with dynamic investment, in which the firm stores a commodity used in production to manage risks generated by productivity shocks and changes in the commodity price. In our model, as in Froot, Scharfstein, and Stein (1993), net worth risk management is motivated by external financing costs: risk is managed to reduce the costs triggered in states in which the firm is financially constrained.

In the model, the operating cash flow is convex in the commodity price, because it is possible to adjust the amount of commodity used in production. Such convexity would suggest an incentive for the firm to increase risk. However, the presence of equity issuance costs generates the opposite

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<sup>1</sup>Chen, Frank, and Wu (2005) document the decline in inventory holdings of U.S. manufacturing firms between 1981 and 2000. Bates, Kahle, and Stulz (2009) found that one of the main factors that caused an increase in American firms' cash holdings was the reduction of net working capital.

incentive to manage net worth risk. The firm shifts net worth between states using inventory: when the commodity price is relatively low, the firm invests in storage and in this way it transfers net worth from states in which the marginal value of net worth is low to states in which it is high. Because the technology in which the firm invests has decreasing returns to scale, the ensuing reduction of net worth dispersion allows investment in capital to be smoothed, yielding an increase in real investment.

We find that inventory management allows the firm to smooth investment in the face of net worth risk, because the value of input inventory has naturally a negative correlation with the cash flow from operations, and that this significantly adds value to the firm. Inventory as a natural hedge is effective even when cash flow risk is mainly generated by productivity (as opposed to commodity price) shocks. In other words, although inventory is a natural hedge against commodity price risk, it proves to be as effective also against other risks affecting the profitability of the firm because it ultimately implements net worth risk management.

Unlike with contingent financial contracts, risk management using inventory is not subject to collateral or margin requirements. This difference has important implications: even though inventory investment is increasing in net worth, risk management is not precluded by low net worth, and absence of risk management occurs only when the marginal value of hedging is low (i.e., when the price of the commodity is high). This is in contrast with the prediction made by Rampini, Sufi, and Viswanathan (2014), who show that low net worth firms do not hedge while high net worth firms do hedge when one relies on contingent hedging tools, for example, derivatives, that require pledging collateral. Hence, our model predicts a positive sensitivity of inventory investment to expected commodity price changes, regardless of the level of net worth. We test this prediction on empirical data from manufacturing firms in the United States and find a positive and significant sensitivity of inventory investment to expected price changes for any level of net worth.

Our analysis offers a novel angle on the traditional comparison between real flexibility (in our case, inventory management) and financial flexibility, and more generally on enterprise risk management. Although the real flexibility of storage has been widely recognized, to the best of our knowledge we are the first to present a financial dimension of inventory other than its simply

being a source of liquidity (as, for instance, in Fazzari and Petersen 1993, Carpenter, Fazzari, and Petersen 1994, Kashyap, Lamont, and Stein 1994).

We then analyze inventory management in conjunction with cash management. While our model shares the same motives that make savings and inventory substitutes, as in Gao (2018) and Kulchania and Thomas (2017) (a transactional motive), and as in Dasgupta, Li, and Yan (2016) (liquidity), it also allows us to show that these motives do not exhaust the set of possible interactions. We actually highlight a complementarity between the two tools: because they are noncontingent, cash holdings can better conserve net worth in future states with low price, in which inventory management will not be effective as a reserve of value. These states, however, are the ones in which the incentive to invest in inventory would be higher. Therefore, savings mitigate underinvestment in inventory due to financing constraints in future states with low price. As a result, the firm optimally invests in inventory and saves, and favors cash holdings when the current commodity price is relatively high. This role of savings is particularly valuable when the commodity has a persistent price and in the presence of adjustment costs that make capital stock and cash non-fungible. Indeed in the model, the value created by inventory is higher if the firm can also save. Moreover, the role of savings is mostly determined by its interaction with the operational hedge.

We test this prediction by analyzing the interaction between storage and savings in the data from manufacturing firms in the United States and find that inventory and cash holdings are both positively correlated to financing constraints and cash flow volatility, which together determine the expected cost of external financing. This result can only be explained by the complementarity between inventory and cash predicted by the model and is thus in contrast with the view that inventory is simply a source of liquidity, whereby cash holdings substitute inventory in the presence of higher risk and tighter financing constraints. Consistent with the model's prediction, we empirically observe that the complementarity is stronger for firms with higher investment rates, in line with the idea that risk management is more important for firms with high financing needs. We also document that capital investment is more sensitive to lagged inventory than to lagged savings, confirming the prediction regarding the role of inventory as a hedge. The dynamic complementarity of the two risk management tools manifests itself along the time-series dimension: everything else

equal, firms with more cash holdings tend to use more extensively the operational hedge offered by inventory.

In this paper, we concentrate on the management of input inventory because it is the main type of inventory, as remarked by Blinder and Maccini (1991).<sup>2</sup> The results regarding risk management in a model based on output inventory would be quite similar to those obtained here, because the flexibility of reducing or shutting down production is limited by frictions related to labor contracts and production set up costs. The difference is that the firm would store finished goods to save on future production costs if it expects higher marginal production costs.

The paper continues as follows. In Section I, we review studies closely related to ours and highlight our contribution to the literature. In Section II, we develop the model and present analytical results regarding the firm's optimal policies and our predictions on the role of inventory in net worth risk management, both in isolation and together with savings. In Section III, we calibrate the model and perform a quantitative analysis of the sensitivity of inventory investment to price changes in relation to the current net worth, and of the complementarity between inventory and savings. Using a sample of American manufacturing firms, we test the model's predictions. Section IV concludes.

## I. Related literature

Our paper relates mostly to the risk management literature, especially with regard to the integrated management of investment and hedging. In our model, risk management is motivated by external equity financing costs. Alternative motivations for risk management can be distress and bankruptcy costs and increased tax payments (Smith and Stulz 1985), tax shield loss (Stulz 1996, Leland 1998), and agency costs (DeMarzo and Duffie 1995).

We study the integrated investment and liquidity policies of the firm, as in Bolton, Chen, and Wang (2011), who recognized the interactions between financial risk management and investment decisions, in their ability to provide liquidity. We also contribute to the literature on the integrated risk management policy, or more generally, on the integration of real and financial flex-

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<sup>2</sup>In our Compustat data, the average ratio of raw materials to total inventory is 38%, and the average input inventory (i.e., raw materials and work-in-process) is 60% of total inventory.

ibility.<sup>3</sup> Gamba and Triantis (2014) found that derivatives are inefficient instruments for hedging real frictions, which can be better managed using cash holdings and production flexibility. Empirical studies, such as Allayannis, Ihrig, and Weston (2001), Pantzalis, Simkins, and Laux (2001), MacKay (2003), and Hankins (2011), support the view that substitution between operational flexibility and financial flexibility is imperfect. Gézcy, Minton, and Schrand (2006) provide evidence of the complementarity between the storage of natural gas and cash holdings in a sample of gas companies in the United States.

In contrast to previous studies, we bring the economics of inventory into corporate finance to explore the role of inventory in risk management. We focus on the flexibility of adjusting storage of production inputs in relation to their market prices,<sup>4</sup> and we analyze the interactions between real flexibility provided by inventory and financial flexibility provided by cash holdings.

Rampini and Viswanathan (2010), Rampini and Viswanathan (2013), and Rampini, Sufi, and Viswanathan (2014), focusing on financing constraints deriving from collateral requirements on debt, provided a rationale for the observed limited use of derivatives by firms with low pledgeable net worth based on the trade-off between debt financing and risk management. In our model, although based on a similar production function with commodity as a production factor as in Rampini, Sufi, and Viswanathan (2014), risk management is implemented using inventory. Because inventory does not require collateralization, it does not tie risk management together with financial policies. Hence, we can abstract from debt financing under limited commitment, and the firm can manage risk and invest at the same time, even when net worth is low.

In line with Rampini and Viswanathan (2010), Nikolov, Schmid, and Steri (2018) showed that financing constraints, in the form of collateral requirements on credit lines and debt financing, determine a motive for holding cash. In relation to this work, we study inventory, a contingent operational hedge which, unlike credit lines, does not require collateral. Therefore, our model predicts that inventory and cash are complementary risk management tools mainly in relation to the support that savings provide for inventory investment, especially when commodity price is low.

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<sup>3</sup>Early studies include Mauer and Triantis (1994) and Mello, Parsons, and Triantis (1995).

<sup>4</sup>The way in which we model inventory management is reminiscent of the theory of storage (see Kaldor 1939, Working 1948, Gustafson 1958, Brennan 1958, Telser 1958), which studies how commodity prices are formed.

Our work relates to the literature that emphasizes the financial component of inventory management. Fazzari and Petersen (1993), Gertler and Gilchrist (1994), Kashyap, Lamont, and Stein (1994) and Carpenter, Fazzari, and Petersen (1994) provided empirical evidence on the liquidation value of inventory, especially for financially constrained firms. Unlike these studies, we highlight a specific channel (hedging of commodity price risk) through which investment in inventory mitigates negative shocks to cash flows. The inclusion of this motive highlights new interactions between inventory and cash holdings leading to a complementarity of the two tools.

We also contribute to the now vast literature on the precautionary motive for holding cash, among others Kim, Mauer, and Sherman (1998), Opler, Pinkowitz, Stulz, and Williamson (1999), Gamba and Triantis (2008), Riddick and Whited (2009), Bates, Kahle, and Stulz (2009), and Bolton, Chen, and Wang (2011). Previous studies have taken a general perspective on cash flow uncertainty and savings, while we disentangle specific sources of cash flow volatility and study how savings respond to each one. We show that focusing on cash flow risk as a whole hides important insights on risk management.

Recent finance literature has shown renewed interest in inventory.<sup>5</sup> Dasgupta, Li, and Yan (2016) found that financially constrained firms vary their inventory stock more aggressively than their financially unconstrained counterparts in response to production costs shocks. Given their assumption that cash holdings and inventory are (imperfect) substitutes, their results would remain basically unchanged if the firm were allowed to hold cash. In contrast, we highlight the role of inventory as a hedge against net worth risk, because it is negatively correlated with cash flow shocks, and show that it can complement savings.

While we focus on the cross-sectional outcome of the dynamic management of inventory and cash holdings, recent studies study how secular technological changes have affected optimal holdings. Gao (2018) contends that the reduction in input inventory stock and the contemporaneous increase in cash holdings for U.S. corporations in recent decades can be explained by the adoption of just-in-time production by a large share of these U.S. firms and the consequent need of cash for transactional purposes. Kulchania and Thomas (2017) showed that the reduction of inventory

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<sup>5</sup>Belo and Lin (2012) and Jones and Tuzel (2013) proposed asset pricing models that predict a negative correlation between inventory investment and stock return risk, consistently with empirical data.



due to deregulation in the trucking industry and the adoption of innovations in supply chain management is an important determinant of the secular increase in corporate cash holdings.

In both studies, what drives the substitution between inventory and cash holdings is the adoption of new technologies over time. We stress that a risk management motive generates positive inventory holdings even in the presence of a just-in-time production system. In our model, the firm faces no delivery lags when it purchases production inputs, which can be seen as an ideal just-in-time system. Still, the firm holds inventory to manage the commodity price risk. At the same time, a transactional motive is also present in our model, because savings can be used to purchase the commodity in states with high price, thus mitigating the impact of external financing costs. However, none of these motives explains the positive relation between inventory and savings that we observe in Compustat firms. The observed complementarity is captured in our model by considering inventory as an operational hedge against net worth risk, and observing that inventory is apt to transfer net worth to future states with high commodity price, whereas savings are more suitable to conserve value in states with low price.

## II. Model

We study a model that presents dynamic investment, production choices, and risk management. The firm is exposed to productivity shocks and commodity price risk for a factor used in production. The impact of input price risk can be reduced by storing the commodity. The need to coordinate investment with cash flows in the presence of external financing costs is the risk management motive in the model.

### A. Firm policies

We consider an all-equity financed firm that faces external financing costs and whose production is based on fixed capital and a commodity as inputs. The manager of the firm, who acts on behalf of the shareholders, maximizes the value of the firm. The shareholders and the manager are risk neutral. We model the manager's decisions in a discrete-time and infinite-horizon setting.

We assume that the firm has a Cobb-Douglas production function,  $g(z_t, k_t, u_t) = z_t k_t^\theta u_t^\gamma$ , in which labor has already been optimized in the current period,  $k_t \geq 0$  is the capital stock,  $u_t \geq 0$  is the amount of commodity used in production, and  $z_t > 0$  is the total factor productivity of the technology at time  $t$ . The parameters  $\theta \in ]0, 1[$  and  $\gamma \in ]0, 1[$  gauge the productivity of capital and commodity, respectively, under a decreasing returns to scale assumption,  $\theta + \gamma < 1$ . The commodity has price  $p_t > 0$  at time  $t$ . The two random variables of the model,  $z_t$  and  $p_t$ , together define the exogenous state of the firm. We assume that  $z_t$  and  $p_t$  have an idiosyncratic component, in addition to a common systematic factor. As for  $p_t$ , while the systematic component is related to the market risk of the commodity, the idiosyncratic component can be interpreted as a supply chain shock. The two variables have compact supports:  $[z_\ell, z_u]$  and  $[p_\ell, p_u]$ , respectively. We assume the joint process  $(z_t, p_t)$  is a Markov chain.

At time  $t$ , in anticipation of a price increase in the coming period, the manager can decide to store away an amount  $n_{t+1} \geq 0$  of commodity. To this aim, she purchases at  $p_t$  an amount  $u_t + i_t^n$  of commodity, where  $i_t^n = n_{t+1} - n_t$  and  $n_t$  is the current stock of commodity. Storage costs of warehousing, deterioration, damages, and theft are  $h(n_{t+1})$  over the period, where  $h(0) = 0$ ,  $h'(n_{t+1}) \geq 0$  and  $h''(n_{t+1}) > 0$ . The strict convexity of  $h(\cdot)$  captures limitations in storage capacity.

In our model, the convenience value of inventory results from the ability to build a stock of commodity when the price is low, and to use the stored commodity in production when the price is high. The avoidance of stockouts, which can be costly if the manager is forced to purchase the commodity at a high price, is part of this convenience value. In the literature on inventory, the (opportunity) cost of stockout is generally determined by lost production. Producing at a high cost gives an equivalent effect of reducing the operating cash flow. We make an important deviation from traditional inventory models (Kydland and Prescott 1982) and follow Humphreys, Maccini, and Schuh (2001) by not including  $n_t$  in the production function, in order to isolate the role of inventory in risk management. Our way of modeling the management of inventory derives from the theory of storage (Kaldor 1939, Working 1948, Brennan 1958). Because we assume risk neutral agents, we ignore the risk premium of investing in the commodity.

At  $t$ , the manager can adjust the capital stock by investing an amount  $i_t^k = k_{t+1} - (1 - \delta)k_t$ . We assume that capital has a constant unit price. Finally, the manager can build financial slack

over the coming period by saving  $c_{t+1}$  in the cash account. Savings are penalized, as they yield a return,  $r$ , lower than that earned in the market by risk-free securities,  $r < 1/\beta - 1$ , where  $\beta$  is the discount factor. The reduced form approach to a penalty on savings is motivated by a wedge between taxation of returns on a firm's savings and taxation of returns on the shareholders' savings. Motivations like free cash flow agency issues or a debt-equity cashing out issue would be inconsistent with the other assumptions of the model.

We summarize the decision variables with  $(k_{t+1}, n_{t+1}, c_{t+1})$ . In this part of the paper, for the sake of analytical tractability, we exclude adjustment costs on capital stock and leave the sign of  $u_t + i_t^n$  unrestricted. In Section III, we consider convex adjustment costs for capital stock and assume that  $u_t + i_t^n \geq 0$ .<sup>6</sup>

Given the current state of the firm and the above described policies, the dividend at  $t$  is

$$d_t = z_t k_t^\theta u_t^\gamma - \psi - p_t (u_t + i_t^n) - h(n_t) - i_t^k + (1 + r)c_t - c_{t+1}, \quad (1)$$

where  $\psi \geq 0$  are fixed costs. The instantaneously optimal amount of commodity is

$$\hat{u}_t = \left( \frac{\gamma z_t k_t^\theta}{p_t} \right)^{\frac{1}{1-\gamma}}. \quad (2)$$

Replacing the expression of  $\hat{u}$  in (1), we can rewrite the dividend as

$$d_t = \pi(z_t, p_t) k_t^\alpha - \psi - p_t i_t^n - h(n_t) - i_t^k + (1 + r)c_t - c_{t+1}, \quad (3)$$

where  $\alpha = \theta/(1 - \gamma) < 1$ , and the net productivity of capital stock is

$$\pi(z_t, p_t) = \gamma^{\frac{1}{1-\gamma}} \left( \frac{1}{\gamma} - 1 \right) \left( \frac{z_t}{p_t^\gamma} \right)^{\frac{1}{1-\gamma}}. \quad (4)$$

Under the assumptions we made on the parameters of the model, the function  $\pi$  is non-negative, increasing and convex in  $z_t$ , and decreasing and convex in  $p_t$ . Therefore, production is negatively affected by a positive shock on commodity price.

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<sup>6</sup>The quantitative effect of this constraint on firm value is quite limited (on average, lower than 4%), as the limitation to sell the commodity increases the demand for external financing.

Although in the dynamic optimization the decision on inventory and production occur on the same date  $t$ , we assume that the manager decides first the current production,  $\hat{u}_t$ , and next  $n_{t+1}$ , given the optimal cash flow from operations. This simplification is possible because the first-order condition on  $u_t$  is independent of  $n_t$ , which is known at time  $t$ .

## B. Firm value optimization

In line with Froot, Scharfstein, and Stein (1993), without loss of generality we assume that capital market imperfections make external equity funds more expensive than those generated within the firm. If other forms of cheaper external fundings, like debt, are considered, our results still apply assuming access to these funds has been already exhausted. Let  $V$  denote the value of the firm as a function of the state  $(z_t, p_t, k_t, n_t, c_t)$ , which results from the maximization of the present value of future dividends with respect to the control variables:

$$V(z_t, p_t, k_t, n_t, c_t) = \max_{(k_{j+1}, n_{j+1}, c_{j+1}), T} \mathbb{E}_t \sum_{j=t}^T \beta^{j-t} [(1 + \lambda)d_t^- + d_t^+], \quad (5)$$

where  $T$  is the stochastic default time, which is chosen based on current information to maximize shareholders' value. This implements the limited liability option of the equity holders: when the value is negative, the firm is abandoned.

In (5),  $d^+ = \max\{d, 0\}$ , and  $d^- = \min\{d, 0\}$ . The cost of external funds, gauged by  $\lambda > 0$ , is the reason why risk is managed: when  $d_t < 0$ , the manager issues equity at a cost  $\lambda d_t$ . The external finance cost can be motivated by taxation, adverse selection, and transaction fees, as summarized by Fazzari, Hubbard, and Petersen (1988). The optimal quantity in (2),  $\hat{u}$ , is unaffected by external financing costs. This is because  $d$  is strictly concave in  $u$ , so there is only one  $\hat{u}$  such that  $\partial_u d(\hat{u}) = 0$ . If  $d(\hat{u}) \neq 0$ , then the conclusion that  $\hat{u}$  is independent of  $\lambda$  is obvious. If, in the opposite case,  $d(\hat{u}) = 0$ , because there is only one maximum,  $d$  is always non-positive and the actual dividend is always  $d(1 + \lambda)$ , which also implies that  $\hat{u}$  does not depend on  $\lambda$ .

We now turn to a recursive description of the model of the firm. Primed variables are referred to time  $t + 1$ , and non-primed variables to time  $t$ . For convenience, we denote the exogenous state variables with  $s = (z, p) \in \mathcal{S}$ , and the decision variables with  $x = (k, n, c)$ . With a little abuse

of notation, we will indicate  $p(s)$  the price  $p$  in state  $s$ . Similarly to Cooley and Quadrini (2001) (and also Hennessy and Whited, 2007, and Rampini, Sufi, and Viswanathan, 2014), we introduce the notion of *realized net worth* at  $s$ , denoted  $w(s, x)$ , which is a sufficient statistic for the state of the firm. Unlike Rampini, Sufi, and Viswanathan (2014), who assume non-negative net worth (e.g., because of a debt covenant) and non-negative dividends, we model strategic default in the sense that, although the firm has negative net worth and can have negative dividends, default does not occur until the cum-dividend value of equity is zero. This is a non-trivial extension of their model, as a non-negative net worth covenant would reduce shareholders' value, thereby reducing the incentive to manage risk. Indeed, an assumption of non-negative dividends corresponds to assuming infinitely costly external financing, which is a separate risk management motive in itself (see, for example, Froot, Scharfstein, and Stein (1993), p. 1635). In relation to Rampini, Sufi, and Viswanathan (2014), we analyze corporate risk management in the absence of both the non-negative dividends restriction and the net worth covenant.

According to the *principle of optimality*, we can solve the manager's program using a recursive approach

$$V(s, w) = \max \left\{ 0, \max_{d, x', \mathbf{w}'} (1 + \lambda) d_t^- + d_t^+ + \beta \int V(s', \mathbf{w}') \mu(ds'|s) \right\}, \quad (6)$$

where  $\mathbf{w}' = (w(s', x'), s' \in \mathcal{S})$ , subject to

$$w(s', x') = \pi(s')(k')^\alpha - \psi + p(s')n' - h(n') + (1 - \delta)k' + (1 + r)c' \quad \text{for all } s' \in \mathcal{S} \quad (7)$$

$$w = d + p(s)n' + k' + c', \quad (8)$$

and  $k' \geq 0$ ,  $n' \geq 0$ , and  $c' \geq 0$ . In (6),  $\mu(\cdot|s)$  is the probability distribution of  $s' = (z', p')$ , conditional on state  $s$ . We assume that  $\mu$  has the Feller property.

The following propositions, which are instrumental to the analysis of corporate risk management in the next section, characterize the value function  $V(s, w)$  and the optimal policy function. The proofs are in Online Appendix A.

**Proposition 1.** *Under the assumption that the distribution  $\mu$  has the Feller property, the value function,  $V$ , of the program (6)-(8) exists, is unique, and coincides with the value function attained solving the program in (5).*

**Proposition 2.** *For each  $s$ , there are three thresholds denoted  $w_d(s)$ ,  $\underline{w}(s)$ , and  $\bar{w}(s)$ , such that  $w_d(s) < \underline{w}(s) < \bar{w}(s)$ . The thresholds define three regions for the realized net worth,  $w$ , which are relevant for firm policy. The firm defaults if  $w \leq w_d(s)$  and is solvent if  $w > w_d(s)$ . When the firm is solvent,*

- a) if  $w \in ]w_d(s), \underline{w}(s)[$  the dividend is  $d = w - \underline{w}(s)$  (i.e., the manager raises an amount  $\underline{w}(s) - w$  of equity) and the adjusted net worth is  $e = w - d = \underline{w}(s)$ ;*
- b) if  $w \in [\underline{w}(s), \bar{w}(s)]$ , there is no dividend and the adjusted net worth is  $e = w$ ;*
- c) if  $w \in ]\bar{w}(s), \infty[$ , the dividend is  $d = w - \bar{w}(s)$  and the adjusted net worth is  $e = \bar{w}(s)$ .*

**Proposition 3.** *The optimal investment policy in capital and inventory and the optimal saving policy, as a function of  $(s, w)$ , is unique and continuous. At state  $s$ , if  $w \leq \underline{w}(s)$  the optimal policy is  $(\underline{k}(s), \underline{n}(s), \underline{c}(s))$ , and if  $w \geq \bar{w}(s)$  the policy is  $(\bar{k}(s), \bar{n}(s), \bar{c}(s))$ .*

**Proposition 4.** *The cum-dividend value function,  $V(s, w)$ , is concave for  $w \geq w_d(s)$  and strictly concave for  $w \in ]\underline{w}(s), \bar{w}(s)[$ .  $V(s, w)$  is continuously differentiable in  $w$ .*

**Proposition 5.** *The optimal investment in capital and optimal investment in inventory satisfy condition*

$$\int \partial_w V(s', w') \left[ \frac{p(s') - h'(n)}{p(s)} \right] \mu(ds'|s) \leq \int \partial_w V(s', w') [\pi(s') \alpha k^{\alpha-1} + (1 - \delta)] \mu(ds'|s), \quad (9)$$

*(where  $w' = w(s', x')$ ) and  $n' > 0$  when the relation above holds as an equality. The left-hand side of (9) is the expected marginal value of inventory and the right-hand side is the marginal productivity of capital. If the commodity price is high, the expected marginal value of inventory is low and it is optimal not to invest in inventory irrespective of the current net worth,  $w$ .*

From Proposition 4, the presence of external finance costs ( $\lambda > 0$ ) determines the concavity of the value function with respect to net worth for low levels of current net worth (i.e.,  $w < \bar{w}(s)$ ). Also in Rampini, Sufi, and Viswanathan (2014) the value function is concave in  $w$ , although the motivation is different. In their case, collateral constraints on debt and derivative contracts are the reason for concavity. In our model, financing constraints in the form of external financing costs, as

in Froot, Scharfstein, and Stein (1993), make  $V$  concave, inducing a demand for risk management as if the shareholders were *risk averse*.

By way of illustration of the above results, Figure 1 shows the optimal policy against the realized net worth,  $w$ , at different  $s = (z, p)$ . We choose the states to represent the impact of a high vs low commodity price for a given productivity (Panel A vs Panel B), and the impact of a low vs high productivity at a low commodity price (Panel C vs D).

In all panels, if the firm is currently solvent (i.e.,  $w > w_d(s)$ ), the optimal policy has a simple form, according to the three mutually exclusive payout regions described in Proposition 2. In the first region,  $w \in ]w_d(s), \underline{w}(s)[$ , the manager issues equity to raise  $d = w - \underline{w}(s)$  at a cost of  $\lambda d$  per unit and achieves an adjusted net worth  $e = w - d = \underline{w}(s)$ . It makes sense for her to raise costly external equity because in this region the marginal value of  $e$  is relatively high. At this point, based on Proposition 3, the manager invests  $e$  in  $(\underline{k}(s), \underline{n}(s), \underline{c}(s))$ , which depends on  $s$  only. Notably, in all states  $s$  it is never optimal to save at  $e = \underline{w}(s)$ , so  $\underline{c}(s) = 0$ , given the benefit is inferior to the cost  $(1 + \lambda)$ . In the second region,  $w \in ]\bar{w}(s), \infty[$ , the firm has excessive net worth, and the manager pays a dividend  $d = w - \bar{w}(s)$  because the marginal value of  $e$  to the shareholders is lower than the marginal value of the dividend. Having achieved the adjusted net worth  $e = \bar{w}(s)$ , the manager invests it in  $(\bar{k}(s), \bar{n}(s), \bar{c}(s))$ , which depends on  $s$  only. In the third region,  $w \in [\underline{w}(s), \bar{w}(s)]$ , the firm is self-sufficient because the marginal value of  $e$  is lower than  $1 + \lambda$  and higher than 1. In this region, the investment, inventory, and saving policies depend on both  $s$  and  $w$ . As stated in Proposition 5 investment in capital always dominates investment in inventory.

To analyze the effect of the commodity price, when  $p(s)$  is high as in Panel A it is optimal not to invest in inventory for any level  $e$ , because the expected marginal benefit of inventory is low and saving becomes the dominant risk management strategy, as predicted by Proposition 5. This decision is based on the current price, irrespective of net worth. On the other hand, if we consider a state characterized by a sufficiently low price, inventory management is positive at all levels of adjusted net worth,  $e$ , and complements cash holdings.

Focusing on states with relatively low price, in which the incentive to use inventory as an operational hedge is positive, the choice of savings vs inventory is determined by productivity,  $z$ . In Figure 1 Panel C, the incentive to invest in capital stock is low and this reduces the opportunity

cost of cash holdings, which is used together with inventory to hedge against net worth risk. Differently from this case, in Panel D a high productivity,  $z$ , makes investment in capital more attractive, and the opportunity cost of savings so high, that savings are crowded out by the more efficient hedge offered by inventory.

### C. Inventory as an operational hedge

From Proposition 4, costly external finance causes the concavity of the value function with respect to  $w$ . Therefore, when firm value maximization is considered, the shareholders are as if they were *risk averse* and there is an incentive to manage net worth risk.

Risk management is value improving because of the concavity of  $V$ . The shareholders achieve a higher value by transferring current net worth to those future states in which the marginal value of net worth is higher (i.e., net worth is lower), thereby reducing the dispersion of net worth. Formally, in (6) given two policies,  $x'_1 = (k'_1, n'_1, c'_1)$  and  $x'_2 = (k'_2, n'_2, c'_2)$ , that induce the same payout,  $d(s, w, x'_1) = d(s, w, x'_2)$ , then  $x'_1$  is preferred to  $x'_2$  if and only if  $\mathbb{E}[V(s', w(s', x'_1))|s] \geq \mathbb{E}[V(s', w(s', x'_2))|s]$ . On the basis of Jensen's inequality, given the concavity of  $V(s, \cdot)$ , this occurs only if the dispersion of  $w(s', x'_1)$  is not higher than the dispersion of  $w(s', x'_2)$ .

The intertemporal mechanism of inventory as a operational hedge can be understood from the optimality condition of investment in inventory. If  $n' > 0$ , from the proof of Proposition 5) we have

$$1 + \lambda \chi_{\{d < 0\}} = \beta \int \frac{1}{p(s)} \partial_w V(s', w') \partial_n w' \mu(ds'|s) = \beta \int [(1 + \lambda) \chi_{\{d' < 0\}} + \varphi(s', w') \chi_{\{d' = 0\}} + \chi_{\{d' > 0\}}] \left[ \frac{p(s') - h'(n')}{p(s)} \right] \mu(ds'|s), \quad (10)$$

where we denote  $\chi_{\mathcal{E}}$  the indicator function of event  $\mathcal{E}$ ,  $\partial_n w' = p' - h'(n')$ , and  $\varphi(s', w')$  such that  $\varphi(s', w') \in [1, 1 + \lambda]$ , is the marginal value of net worth when the firm is self-sufficient (i.e.,  $d' = 0$ ). The current cost to change inventory, in the left-hand side of (10), is augmented by the possible equity issuance cost. This must be equal to, on the right-hand side, the expectation of marginal value of inventory in state  $s'$ ,  $(p(s') - h'(n'))/p(s)$ , times the marginal value of net worth. Ultimately, the value contributed by inventory management is the reduction of the equity issuance



costs at  $t + 1$ , obtained by transferring net worth from states in which the marginal value of net worth is low to states in which it is high.

The way this operational hedge works is simple: a high input price reduces production and net worth, and the manager insures against a reduction of cash flow in those future states in which the commodity price is high by investing in inventory in the current period. Specifically, when  $p(s)$  is low (i.e., the marginal value of inventory is high) as in Figure 1, Panels C and D, the manager invests in risk management at a cost  $p(s)n'$ , if necessary reducing savings and current investment in capital stock. The effect of this decision is to increase the net worth in state  $s'$  by  $p(s')n' - h(n')$ . This effect will be larger in those future states in which  $\pi(s')$  from equation (4) is lower precisely because  $p(s')$  is higher.

It should be noted that, in the model, there is no speculation or profit making motive for managing inventory: given the assumption  $n \geq 0$ , the firm cannot take a short position in inventory that would allow profiting on a decrease of  $p$ . Also, the fact that for analytical tractability  $u_t + i_t^n$  can be negative in some states does not imply the manager can speculate on the commodity. Selling part of the unused stock of commodity at the current price affects only the amount of cash available to the firm (i.e., the risk management incentive), not its profit.<sup>7</sup>

## D. Inventory and cash holdings

In the presence of commodity price risk, the main difference between cash holdings and inventory is that the latter is a contingent risk management tool, whereas the former is noncontingent. Cash holdings, by yielding a risk-free return, although reduced by the deterministic cost  $1/\beta - (1 + r)$ , decrease the probability of equity issuance in all states  $s'$ . The downside is that savings are inefficiently held in states of the world with high net worth (i.e., a high opportunity cost of cash).

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<sup>7</sup>We do not consider derivatives in our baseline model. In Online Appendix C, we describe the similarities and differences between derivatives and inventory, and show that the inclusion of derivatives in the integrated risk management policy would not affect our main conclusions.

This is clear from the optimality condition for the cash holdings decision (see proof of Proposition 5), if  $c' > 0$ ,

$$1 + \lambda \chi_{\{d < 0\}} = \beta \int \partial_w V(s', w') \partial_c w' \mu(ds'|s) = \beta (1 + r) \int [(1 + \lambda) \chi_{\{d' < 0\}} + \varphi(s', w') \chi_{\{d' = 0\}} + \chi_{\{d' > 0\}}] \mu(ds'|s), \quad (11)$$

where  $\partial_c w' = 1 + r$ . On the left-hand side, savings may entail external financing costs if the manager is in an equity-issuance state.

Allowing both inventory and savings, if  $\sigma_p = 0$  also inventory becomes noncontingent and contrasting the right-hand side of (11) with the right-hand side of (10), the manager substitutes inventory for savings because storage costs are non-negligible and savings yield a non-negative return. Conversely, if  $\sigma_p > 0$ , the incentive to hold inventory as an operational hedge becomes positive, because of the positive probability of states  $s'$  in which the operating cash flow is low and the inventory value is high.

A simple comparison of the first order conditions (11) and (10) shows that inventory and savings are substitutes at the margin, in the sense that the manager will choose to invest an additional dollar of adjusted net worth,  $e$ , in the one of the two with the highest marginal benefit. In some extreme states, like the ones in Panels A and D of Figure 1, the substitution is complete, as opposed to just marginal. Still, from the analysis of the optimal investment policy, savings and inventory coexist, as shown in Panels B and C of Figure 1.

We contend that inventory and cash holdings can be complementary risk management tools in some states. This is because financing constraints induce underinvestment in inventory, and savings can increase the risk management capacity (using inventory) of financially constrained firms in future states in which the commodity price will be low. Cash holdings are better suited to sustain risk management in states with low  $\pi(s)$  and  $p(s)$ , as in Panels B and C of Figure 1. In such states, the manager partly saves instead of using all the residual resources for inventory, because a persistent price will likely be low also next period, and net worth will be better conserved using savings. More generally, besides using inventory to transfer net worth to states  $s'$  with high price, by saving a firm can efficiently transfer net worth to states with low benefit,  $(p(s') - h'(n'))/p(s)$ ,

offered by inventory. In light of this complementarity, coexistence of inventory and cash holdings is to be expected—a scenario like the one in Panel A of Figure 1 being an extreme case, in which on average the benefit of inventory is so small that savings become dominant. The complementarity is even more important in the presence of adjustment costs, at the moment excluded, which render capital stock and cash holdings non-fungible.

To summarize the main intuition derived from Figure 1, inventory is a valuable risk management tool. With costly external financing, a firm with a high cash flow today will save some of it rather than pay it out fully to shareholders. As the firm saves using cash and inventory, when the input price is low or when productivity is high, the firm chooses to hold some inventory. In many states, cash and inventory are complements, and both increase with net worth. With high input prices, cash dominates inventory, and with low input prices and high productivity, inventory dominates cash.

In the literature, inventory and cash holdings have been only considered substitutes because of a transactional motive, which is present also in our model: when the commodity price is expected to increase, the firm can alternatively increase the stock of inventory or cash holdings. With inventory, the firm reduces its exposure to expected external finance costs caused by commodity price risk. With savings (and efficient supply chain management), the firm will be able to purchase the commodity at the spot price when needed, with lower incidence of external finance costs. A transactional motive for holding cash has been studied by Gao (2018) and Kulchania and Thomas (2017), who provide evidence of the substitution between inventory and cash holdings, given the secular adoption by firms of more efficient supply chains. However, this is only part of the joint effect of inventory and cash holdings, because even if cash holdings can reduce external financing costs, the operating cash flow is negatively affected by a positive shock in the commodity price (in other words,  $\pi(s')$  is low when  $p(s')$  is high), and inventory can offset the cash flow shortfall, as illustrated above, whereas savings can be used but are less efficient. Therefore the interaction of savings and inventory goes beyond a mere transactional motive.

A similar argument applies also with reference to the literature (e.g., Dasgupta, Li, and Yan 2016), that sees inventory as a source of liquidity tailored to relax financing constraints. In that setup, for a financially constrained firm exposed to higher net worth risk, cash holdings would

substitute inventory as a less costly—and therefore more efficient—way of holding a reserve of liquidity in the firm. This rationale for substituting inventory with savings is also present in our setup, as storage costs make inventory more expensive to hold than cash. However, this view does not consider that inventory is a contingent tool whose main benefit resides on being negatively correlated with operating cash flow.

To conclude, the analysis in this section shows qualitatively how cash holdings are an important source of financing for risk management for constrained firms. Whether cash holdings and inventory are prevalently complements or substitutes is a quantitative question. Therefore, in Section III, we will resort to a quantitative analysis of the model to show if and when cash holdings’ support to inventory management significantly increases the enterprise value of the firm, and if the ability to save reduces or increases the use of inventory as an operational hedge.

### III. Quantitative analysis

In this section, we calibrate the model so that it replicates empirical moments of a sample of Compustat manufacturing firms. Next, we use the model to formulate empirical predictions regarding inventory investment and the integrated management of inventory and cash holdings. In particular, we focus on the results that risk management using inventory is not reduced when net worth is low, and that cash holdings complement inventory in managing commodity price risk. Finally, we confirm these predictions with empirical findings.

#### A. Calibration

We choose the base case values of the parameters by contrasting moments of variables obtained from a Monte Carlo simulation of the model (see Online Appendix B for details) with their empirical counterparts in the U.S. economy. We construct the empirical sample by starting with all firms in the Compustat North America database in the manufacturing industry (SIC codes 2000-3999), which is a suitable industry given our assumptions on the firm’s technology and the relevance of inventory for these firms. We exclude firms with less than two observations, firms-year observations with negative values of total assets, sales, and book equity; and firms whose book items did not

comply with standard accounting identities. The final sample is an unbalanced panel of firms observed between 1969 and 2014 with at least 1367 observations per year.

We compute the investment rate in capital stock as capital expenditures (capx in Compustat) less sale of capital (sppe) scaled by beginning-of-period capital (ppeg). Inventory investment is computed as the change in stock of raw materials (invm) between two consecutive years scaled by beginning-of-period capital. The inventory ratio is computed as inventory scaled by the beginning-of-period capital or by total assets (at), and the cash ratio as cash and short-term securities (che) scaled by total assets (at). The market-to-book ratio is computed as the sum of total assets and the market value of equity (fiscal year end stock price multiplied by the number of shares outstanding,  $prcc \times csho$ ) minus the book value of equity (ceq) and deferred taxes (txdb), scaled by total assets.

The frequency of equity issuance refers to net sales of stock to which we apply a screening based on a measure of firm-initiated issuances developed by McKeon (2015). The measure is defined as the sale of common equity net of the issuance of preferred equity ( $sstk - prstk$ ) scaled by the equity market value. To capture firm-initiated issuances, McKeon suggests to consider only issuances for which the measure is larger than 3%. In this case, we obtain an issuance frequency of 18%, while if we consider a 5% threshold the frequency reduces to 16%. If we do not exclude preferred equity, the frequency is again in the range 16-18%.

We winsorize all variables at the 1% level to mitigate the impact of outliers. In addition, we compute the probability of inventory divestment considering only observations of negative inventory investment larger than 1% of the beginning-of-period capital stock. Such thresholds are used to reduce the impact of measurement errors. This motivation is particularly relevant for inventory investment, which is typically a small fraction of a firm's assets, and therefore more prone to accounting errors or misreporting.

The base case parameter values are summarized in Table I, while moments are reported in Table II. We calibrate our model in annual frequency, setting the discount factor at  $\beta = 1/1.05$  and the interest rate on cash holding  $r$  at 0.0462.

Productivity shocks are typically modeled as autoregressive processes in the financial economics literature (e.g., Gomes 2001, Hennessy and Whited 2005, Zhang 2005). The productivity has dynamics  $\log z' = \phi_z \log z + \sigma_z \varepsilon'_z$ , where  $|\phi_z| < 1$ ,  $\sigma_z > 0$ , and  $\varepsilon'_z$  are i.i.d. shocks with truncated

standard Normal distribution. In commodity markets, prices tend to revert to the average marginal cost of production (see Schwartz 1997). The commodity price follows the process  $\log p' = \phi_p \log p + \sigma_p \varepsilon'_p$ , where  $|\phi_p| < 1$ ,  $\sigma_p > 0$ , and  $\varepsilon'_p$  are i.i.d. shocks with truncated Normal distribution. The support of  $z$  and  $p$  must be compact to ensure that the dynamic program we described has a solution, and we achieve this by truncating the distribution of  $z$  and  $p$  to within three times the unconditional standard deviation around the unconditional average. More details are given in Online Appendix B. The shock  $\varepsilon_z$  is contemporaneously correlated with  $\varepsilon_p$  so that  $\mathbb{E}[\varepsilon_z \varepsilon_p] = \rho$  and  $\mathbb{E}[\varepsilon_{z,t} \varepsilon_{p,s}] = 0$  for  $t \neq s$ . As mentioned in Section II.A, we assume that  $z$  and  $p$  have an idiosyncratic and a common systematic component, and capture this fact by letting the correlation,  $\rho$ , be between 0 and 1. We set  $\rho$  to 0 for the base case. Assuming  $z$  is systematic, this corresponds to the assumption that the shocks to the commodity price are entirely firm-specific. Given the importance of the systematic component of the stochastic evolution of  $p$  for our results, we will provide comparative statics on  $\rho$  later on.

The autoregression coefficient of  $z$ ,  $\phi_z$ , is set to 0.62 and the volatility  $\sigma_z$  to 0.20, in line with values selected by Gomes (2001) and the estimates of Hennessy and Whited (2005). The autoregression  $\phi_p$  and the volatility  $\sigma_p$  of the commodity price process are also set at 0.62 and 0.20, respectively, in order to have a marginal distribution of  $p$  comparable to that of  $z$ , so that the relevance of risk management using inventory is not overstated. To enable a comparison with real data, we take the time series of the main commodity price indexes from the World Bank GEM Commodities database. Our choice of  $\sigma_p$  is in line with the volatilities of indexes returns. The volatility of agricultural and metals indexes returns is 0.10 and 0.16 respectively, whereas, for the energy index, it averages at around 0.35. Because we are not considering a specific commodity, a value of 0.20 seems reasonable and in line with the values reported by Geman (2005).

As for the production function, in a sample of U.S. industries, Basu (1996) reported an empirical average of 0.60 for the share of materials (our “commodity”) in the total cost of production, while the remaining share is split between capital and labor. We set the overall return to scale in production to 0.90 based on the estimates of Basu and Fernald (1997), and assign a share of productivity of  $\gamma = 0.54 = 0.90 \times 0.60$  to the commodity, and of  $\theta = 0.36 = 0.90 \times 0.40$  to capital. The way we assign productivity shares to factors reflects a production function with commodity and value added specified as production factors, as in Basu (1996). Although, relative to the more

general functional form presented by Basu (1996), for simplicity, we consider a production function after labor has been optimized out. We take labor's productivity into account implicitly by selecting appropriate values for capital and commodity productivity parameters.

We set the capital depreciation rate,  $\delta$ , to 0.12 (e.g., see Gomes 2001) to match the empirical average capital investment rate. In order to obtain the volatility of the capital investment rate and the probability of negative investment in capital close to the respective empirical counterparts, we introduce adjustment costs for capital of the form adopted in the literature on the  $q$ -theory of investment (see Hayashi 1982). When the firm adjusts the capital stock, it incurs adjustment costs

$$a(k, k') = \frac{\xi}{2k} \left( i^k \right)^2, \quad \text{with} \quad \xi = \bar{\xi} \chi_{\mathcal{I}^+} + \underline{\xi} \chi_{\mathcal{I}^-},$$

where  $\mathcal{I}^+ = \{i^k > 0\}$  is the event of investment,  $\mathcal{I}^- = \{i^k < 0\}$  is the event of disinvestment, and  $\bar{\xi} < \underline{\xi}$  gauges partial irreversibility, as in Zhang (2005). As usual, partial irreversibility can be motivated by adverse selection (Arrow 1968), limited assets redeployability (Williamson 1988), or leverage of potential buyers (Shleifer and Vishny 1992, Asquith, Gertner, and Scharfstein 1994). The adjustment cost,  $a(k, k')$ , reduces the dividend,  $d$ , as it is subtracted from the right-hand side of equation (3). We set  $\bar{\xi} = 0.75$  and  $\underline{\xi}$ , so that  $\bar{\xi}/\underline{\xi} = 1/10$ , as in Zhang (2005).

We specify the storage cost function as  $h(n) = (\eta/2)n^2$ , setting  $\eta = 0.034$  to match the average and volatility of inventory scaled by the beginning-of-period capital stock. The ratio of inventory to capital is an appropriate benchmark in relation to inventory management, given the purpose of mitigating the commodity price risk that affects cash flow, which is proportional to capital stock. Our storage cost function is similar to that used by Blinder (1986), with the exception that we restrict the linear and fixed cost components to zero.

Given the values selected for the parameters in the production function, we set  $\psi = 0.03$  and  $\lambda = 0.05$  to approximately match the average market-to-book ratio and the empirical probability of equity issuance. We select a value of  $\lambda$  very close to the estimate (0.058) of Hennessy and Whited (2005), who adopted a linear specification for the equity issuance cost function, like ours. Finally, the joint distribution selected for the productivity shock and for the price of the commodity and the values assigned to  $\lambda$  and  $r$  help to approximate the average and the volatility of cash scaled by total assets in empirical data.

## B. Risk management using inventory

Froot, Scharfstein, and Stein (1993) posited that net worth risk is managed with a view to invest in capital stock when the productivity is high and external finance is costly. This is what we investigate in our model, in which the manager has an incentive to manage risk because the value of equity is a concave function of net worth.

First, we use the calibrated model to understand how important risk management is in our setup, and in particular how much value is created by inventory as an operational hedge. In Figure 2, we plot against  $(z, p)$  the incremental value due to inventory management in a model in which the firm is not allowed to manage cash holdings. We choose as the current state of the firm the unconditional average of  $k$  and  $n$ . It is immediately clear that risk management adds significantly to firm value, and this addition is largest in states in which the firm is relatively less productive (low  $z$ ) and the commodity is more expensive (high  $p$ ). It turns out that inventory is the main risk management tool in our setup, because the value increase is only slightly higher (0.3%-0.8% higher) if the firm can also manage cash holdings. We will further investigate the role of cash holdings together with inventory later on.

Oi (1961) showed that a firm may act in a risk loving manner if the cash flow function is convex in output or input prices. In the case of variable production factors, such convexity derives from the flexibility of optimally adjusting the quantity of the factor to its price.<sup>8</sup> Also Adam, Dasgupta, and Titman (2007) predict that the convexity of the cash flow function with respect to the output price generates an incentive not to hedge, or even to increase risk. This happens also in our model: the operating cash flow is a convex function of the commodity price,  $p$ , because the manager optimizes the use of commodity in each period. However, this reasoning ignores the concavity of the value function induced by financing constraints, as derived in Proposition 4. Therefore, a natural question

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<sup>8</sup>Assuming a production function with constant returns to scale, Hartman (1972) showed that both investment and value increase in the volatility of factor prices as long as the increase in the marginal costs of investment is lower than increase of expected marginal cash flow. Pindyck (1982) showed that a convex adjustment cost function leads to a higher current investment in a dynamic setup, because the volatility of the output or factor prices makes expected future adjustment costs higher, creating a “precautionary” motive for investing earlier. In this case, firm value is reduced because of the higher average cost of production induced by a higher average capital stock for a given price of the output. Abel (1983) showed that the shadow price of capital does not depend on adjustment costs. Therefore, if the marginal product of capital is convex in prices, higher uncertainty leads to higher investment and higher firm value.



is whether the effect pointed out by Oi (1961) dominates the risk management incentive. This is a quantitative question, and therefore we resort to the calibrated model to address it.

In Figure 3, optimal investment and the value of the firm, at the steady state, are plotted against current net worth for different values of  $\sigma_p$ . Varying  $\sigma_p$  changes both risk and the unconditional average of cash flow  $\pi(s)k^\alpha - \psi$ . To offset this effect, we adjust the average values of  $p$ . We keep the unconditional average of cash flow constant, rather than the average net worth, to examine the impact of risk on optimal policies before the effect of risk management on net worth. For the moment, we focus on the effect of inventory and exclude cash management, although the outputs are substantially the same when considering also cash holdings. First, we must consider the case without risk management ('no inventory'). In Panel A, the effect of higher commodity price risk on investment in capital stock is clearly negative. This is because the technology in which the firm invests has decreasing returns to scale, and higher volatility of investment reduces the incentive to invest. The effect that a higher volatility of  $p$  has on reducing investment is a risk management motive, as pointed out by Froot, Scharfstein, and Stein (1993). In Panel B, the effect of higher risk on the value of the firm is positive: the impact of convexity of the operating cash flow with respect to  $p$  dominates the one due to the concavity of  $V$  with respect to  $w$ , when the value of the firm is considered. Overall, in relation to commodity price risk, although the firm appears to be risk loving from the value perspective, there is an incentive to manage risk when investment is considered.

Allowing the firm to use the operational hedge, Panel C ('inventory'), increases investment, and this real effect adds value to the firm at all levels of net worth (Panel D). Because a speculation purpose for managing inventory has been ruled out in the model, the higher value of the case with risk management relative to the case without risk management can only be attributed to higher real investment and lower expected external financing costs. The effect on  $k'$  is remarkable: the ability to manage commodity price risk using inventory allows the firm to reduce the dispersion of net worth and therefore increase real investment.

To some extent, one may consider obvious that inventory is a natural hedge against commodity price risk. However, the focus of the paper is on net worth, as opposed to price, risk. Therefore, one wonders if inventory management can hedge against risks other than price risk, which in our

setup are summarized by shocks on firm’s productivity. Figure 4, similarly to Figure 3, presents investment and firm value for the cases with vs without inventory management. Differently from before, we now consider a comparative statics on  $\sigma_z$ . As before, we adjust the average values of  $z$  or  $p$  when varying  $\sigma_z$  or  $\sigma_p$ , respectively, to keep the unconditional average of cash flow constant. In Panel A, an increase in  $\sigma_z$  reduces investment in capital, for the reason –described above– that the incentive to invest becomes more volatile, in line with Froot, Scharfstein, and Stein (1993). Higher  $\sigma_z$  increases the value of the firm, as can be seen in Panel B. Again, this is a consequence of the convexity of the payoff function.

When we introduce inventory, we see that the effect of using the operational hedge is to increase investment (Panel C) and the value of the firm (Panel D). Therefore, inventory as a tool to manage net worth risk has a significant role also when the main source of risk is not the commodity price, although comparing Figure 4 with Figure 3, the effect of inventory is naturally more sensitive to  $\sigma_p$  than to  $\sigma_z$ .<sup>9</sup>

### C. The sensitivity of inventory to commodity price

Rampini, Sufi, and Viswanathan (2014) tested whether the incentive to manage risk using derivatives would be smaller the lower the current net worth, as predicted by their model. Our research complements their analysis by showing that, empirically, firms engage in operational hedges that do not require collateral (like inventory) even when their current net worth is low when the expected marginal value of inventory is sufficiently high.

Unlike derivatives, inventory does not tie risk management with financial policies. Therefore, it can also be used by firms with less pledgeable (i.e., intangible or firm-specific) assets and does not reduce debt and financial hedging capacity: if anything, inventory increases the collateral value of assets. Firms with low net worth abstain from using derivatives, as shown by Rampini, Sufi, and Viswanathan (2014), but may not abstain from risk management implemented using other approaches (e.g., operational hedging using inventory). Indeed, in our Compustat sample, firms in the lowest tercile of the distribution of net worth hold an average inventory of 35% (or 33%, depending on the proxy used for net worth) of capital.

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<sup>9</sup>In Online Appendix D, we provide also a comparative statics analysis on the effect of the persistence parameters  $\phi_z$  and  $\phi_p$  on the optimal investment policies.

In this section, we test the hypothesis derived from the analysis in Section II.C, that inventory investment is sensitive to the expected commodity price change regardless of the level of net worth. In particular, we test whether low net worth firms do not reduce the effort of managing risk. As we have seen in Figure 1, the model predicts that the sensitivity of inventory investment is not zero, even for low net worth firms, when the expected marginal value of inventory is high.

### C.1. Predictions of the model

We make the hypothesis more precise by estimating the sensitivity of inventory investment to the expected change in the commodity price, conditional on current net worth  $w_t$  in a simulated economy obtained from the calibrated model. Specifically, we estimate the relation

$$\frac{p_t i_t^n}{k_t + p_t n_t} = b_1 \frac{\mathbb{E}[p_{t+1}|p_t] - p_t}{p_t} + b_2 \frac{p_t n_t}{k_t + p_t n_t} + b_3 \frac{V_t}{k_t + p_t n_t} + b_4 \frac{\pi_t k_t^\alpha - \psi}{k_t + p_t n_t} + \epsilon_{t+1}, \quad (12)$$

which holds for each firm-year observation, and where  $\epsilon_{t+1}$  is an i.i.d. error. To obtain a direct comparison with empirical results to be shown later on, we estimate (12) on demeaned variables using OLS, therefore we do not estimate an intercept. Our attention is on the coefficient  $b_1$  related to expected price change, which gauges the risk management incentive to invest in inventory. In (12), current inventory provides information on the magnitude of the marginal benefit of the incremental quantity of commodity stored. Market-to-book ratio proxies for the overall investment opportunities of the firm and cash flow proxies for the availability of internal funds. Our results are the same if we include the management of cash holdings in the model.

The OLS regression coefficients are reported in Table III. The first column of the table presents the regression on the full sample, while the remaining four columns report the estimates on net worth quartile groups of firm-year observations. For the full sample estimates, we enlarge the set of explanatory variables by adding an indicator variable equal to one if the firms net worth is below the median and zero otherwise, and an interaction term between the expected price change and the indicator variable. In this way, the sensitivity of inventory investment to price changes for low net worth firms can be directly calculated as the sum of the regression coefficient related to price change and the coefficient on the interaction term.

In the model, *regardless of the level of net worth*, expected price change positively influences inventory investment given the incentive to manage commodity price risk. Inventory investment is negatively related to current inventory because of a decreasing marginal benefit of holding inventory. The sensitivity of inventory investment to market-to-book ratio is positive because firms with valuable investment opportunities need more hedging to reduce the volatility of net worth and therefore to sustain investment in all states, as described in Section II.C. Although cash flow enlarges the firm’s budget constraint, it is negatively related to inventory investment. This happens because cash flow is a function of stochastic factors that determine investment opportunities.

Our results are robust to using investment in physical (as opposed to dollar-value) of inventory as dependent variable, to scaling variables with capital  $k$  only, and to using different parameter values. We do not report these results for brevity, but they are available upon request. Varying the model parameters, the sensitivity of investment in inventory to expected price changes in the simulated economy is always positive and significant. Interestingly, the sensitivity increases when  $\sigma_z$ ,  $\sigma_p$ , or  $\phi_z$  is higher, or when  $\phi_p$  is lower, suggesting that there is a higher sensitivity of inventory investment to expected price changes when there is a stronger incentive to manage risk. Finally, we vary also the returns to scale parameters given their impact on the exposure of net worth to the commodity price and find results similar to those in Table III.

In Table IV, we study how estimates of (12) change according capital investment (capital investment scaled by the beginning-of-period capital) and cash flow volatility, which are firm characteristics relevant for risk management. As for capital investment, we sort firms into two subsets, one above and one below the median. As for cash flow volatility, we simulate the model for different values of  $\sigma_z$ . While the results are qualitatively similar to those in Table III, the sensitivity to price change is 1.81 (equal to 1.39 plus 0.42) for low net worth firms with high capital investment, as opposed to a 1.05 (1.15 minus 0.10) for firms with low capital investment. The sensitivity to price change is 1.67 (1.25 plus 0.42) for high risk firms, relative to 0.95 (1.39 minus 0.44) for low risk firms. Overall, Table IV shows that when there is a stronger incentive to manage risk because of investment or because of cash flow risk, inventory investment is more sensitive to price changes for firms with low net worth.

## C.2. Empirical test of the predictions

We test the predictions of the model by estimating the relation in (12) using the Compustat sample of manufacturing firms. The results are reported in Table V. In our analysis, inventory investment is the change in stock of raw materials inventory between two consecutive periods (invrmm minus lagged invrm in Compustat). We compute the expected price change using the Producer Price Index (PPI) retrieved from the FRED database and assuming an autoregressive model for the time series of price changes. Although expected price change does not convey information on expected prices at the firm level, it is much less affected by endogeneity because it is an economy-wide indicator. Cash flow is EBIT (ebit), and net worth is the book value of stockholders' equity (seq).

The structure of Table V is similar to that of Table III. All variables, except price change, are scaled by the beginning-of-period total assets (lagged at in Compustat). We use the Erickson, Jiang, and Whited (2014) (EJW) third-order cumulant estimator to address endogeneity caused by measurement error in the market-to-book ratio. For brevity, we do not report estimates obtained when we do not control for such measurement error, but we will discuss the relevant differences, later on.

The results in Tables V are consistent with those in Table III. In line with the role of inventory as a risk management tool, the sensitivity of inventory investment to expected price change is always positive and significant. More importantly, the regression on the whole sample clearly shows that the interaction terms are significant and negative but not sufficiently large in magnitude to cancel the positive sensitivity to price change. This confirms the intuition offered in Figure 1 and made precise by the estimate of  $b_1$  in Table III. Low net worth firms may have limited financial hedging capacity, but they can use an operational hedge like inventory. Notwithstanding a stricter budget constraint, inventory investment for firms with low net worth is positively related to price change. Although we cannot exclude the effect of net worth through the budget constraint, we find that such effect is not sufficiently strong to cancel the incentive to invest in inventory when commodity prices are expected to increase.

The coefficients of the other regressors have signs in line with economic intuition. We find a negative coefficient for current inventory and a positive one for market-to-book. There is a non-negligible improvement in estimates when we control for measurement error in the market-to-

book ratio. Indeed, if we do not use the EJW estimator, in unreported results we find that the sensitivity of inventory investment to market-to-book is smaller (around 0.003) and, considering the full sample,  $R^2$  decreases from 18-19% to 12-13%. The sensitivity of inventory investment to cash flow, in general, is positive and significant for low net worth firms, and negative and less significant for firms with high net worth. In addition, such sensitivity decreases with the net worth. The sign of cash flow sensitivities of investment is not robust because it varies from one empirical setting to the other. Among others, Erickson, Jiang, and Whited (2014) found on average a negative and non-significant sensitivity of investment to cash flow. Poterba (1988) pointed out that cash flow and investment can be spuriously positively correlated because both cash flow and Tobin's  $q$  are correlated with true marginal  $q$ . Although posited for the case of capital investment, Poterba's argument also applies to our case of investment in inventory. Again, if we do not use the EJW estimator, the cash flow sensitivity is increased (considering the full sample, from 0.04 to 0.07) because of the bias induced by measurement error in the market-to-book ratio.

We conduct a series of robustness tests on the empirical regressions. First of all, we use the proxy of net worth based on market values constructed by Rampini, Sufi, and Viswanathan (2014) and obtain similar results. Second, results are not significantly changed when we take into account unobserved industry-specific heterogeneity by including industry fixed-effects in the regressions. Third, we use the lagged realized price change in place of the expected price change and found equivalent results. Fourth, we test the robustness of the proxy of expected price change by replacing it in the regression with returns on indexes of the agricultural, energy, and metals commodities and found similar results. The sensitivity to returns on indexes is significant and *positive* for the agricultural and energy indexes, while it is significant and negative for the metal index. All the interaction terms are positive and significant. Only the coefficient related to the interaction between the indicator and the energy index is negative and significant but not sufficiently large in magnitude to cancel the positive sensitivity of inventory investment to this index. We further test the robustness of expected price change by adding in the regression the real GDP expected growth in order to establish that expected price change is not proxying for economic activity in general. We compute the expected GDP growth based on an AR(1) specification using data from the FRED database. We obtain results equivalent to those reported for the baseline specification. The sensitivity to expected GDP growth is 0.23 and significant at the 1% level, and the magnitude

and significance of the sensitivity to price change remain essentially the same after the addition of GDP growth.

As an additional robustness test, because inventory investment is measured in dollar values, and to make sure our results are not driven by mechanical changes in prices, we deflate inventory using the PPI index and run the same regression using expected returns on commodity indexes in place of expected change in PPI. In this case, the coefficient related to the agricultural index is always positive and significant, while the one related to the energy index is positive but not significant. The sensitivity to the metals index returns is always negative and significant. All the interaction terms are either positive or not significant. Finally, Dasgupta, Li, and Yan (2016) show that the use of LIFO or FIFO accounting, whereby current inventory is valued at different possible prices, does not influence their results on inventory investment. This is also true in our case.

As we did with the model, we estimate (12) sorting the Compustat sample with respect to capital investment (capx minus sppe scaled by ppeg in Compustat) and cash flow (ebit) volatility.<sup>10</sup> We sort firms into two subsets divided by the median of the firm’s characteristic. We report results in Tables VI. While the results are qualitatively similar to those reported in Table V, we find confirmation of the prediction that, for low net worth firms, inventory investment is more sensitive to price changes when capital expenditures are high and when cash flow risk is high. In Table VI, for low net worth firms, the sensitivity increases from 0.03 (0.15 minus 0.12) to 0.10 (0.17 minus 0.07) with capital investment, while it increases from 0.07 (0.20 minus 0.13) to 0.11 (−0.04 is insignificant) with cash flow volatility. These results support the idea (in line with Froot, Scharfstein, and Stein 1993) that the incentive to use inventory as a hedge is bigger for firms with higher external financing needs and higher risk, for which risk management is more valuable.

## D. Integrated risk management with inventory and cash holdings

We analyze the use of inventory and cash holdings in relation to cash flow risk and financing constraints, which together determine the probability of the firm’s need of external finance in the

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<sup>10</sup>We compute firm’s cash flow volatility as the standard deviation of cash flow scaled by total assets over the entire time span in which the firm is observed. Opler, Pinkowitz, Stulz, and Williamson (1999) computed firm’s cash flow volatility over the previous 20 years for each firm-year observation in their sample. However, their method is not suitable for our sample, because it would drastically reduce the number of observations. To test the robustness of our conclusions, we compute cash flow volatility using a five-years window and obtain results (which are available upon request) similar to those reported.

model. In Section II.D, we derived the prediction that risky and financially constrained firms have a bigger incentive to manage commodity price risk using inventory, and that cash holdings can complement the operational hedge to transfer net worth to future states and be used to finance risk management, in addition to substituting inventory as a risk management tool. We hypothesized that such complementarity between inventory and savings is prevalent in the presence of external financing costs and of capital adjustment costs, which render capital stock and cash holdings non-fungible.

### D.1. Complementarity of savings and inventory in the model

We make this prediction quantitatively precise by using the calibrated model to assess whether the incremental value generated by savings is higher with or without inventory. Figure 5 shows the incremental value of cash holdings as a function of the productivity shock  $z$  and the commodity price  $p$ , in a state in which capital  $k$ , inventory  $n$ , and cash holdings  $c$  are at the unconditional average, computed as described in Online Appendix B. First, we analyze the case in which cash holdings are the only risk management tool. In Panel A, we plot  $V^c/V^0 - 1$ , where  $V^c$  is the enterprise value (i.e., net of current savings) in the model with dynamic cash management and no inventory, and  $V^0$  is the value in the model with no inventory and no cash holdings. In Panel B, we compute the incremental value due to cash management in the presence of inventory,  $V^*/V^n - 1$ , where  $V^* = V - c$  is enterprise value and  $V^n$  is the value of a firm that can manage inventory but is not allowed to hold cash.

The value of cash holdings in our setup is by construction related to the presence of external finance costs, as it is for all the other risk management tools. When considered by itself, in Panel A, the incremental value of cash holdings, in terms of our calibration, is quite limited and is higher in circumstances in which the current cash flow is low because  $p$  is high and  $z$  is low. Panel B shows that the incremental value of cash holdings is mainly due to the synergy with inventory. In the presence of external financing costs, even when there is a positive incentive to use inventory as an operational hedge, savings complements the ability of inventory to transfer net worth to future states when inventory is less effective.



In Panel B, the value of the synergy between inventory and cash holdings is always positive and it increases in the commodity price for high values of  $z$ , while it decreases in  $p$  when  $z$  is low. This is because, when productivity is high, there is a high incentive to invest in capital and the realized net worth is high. In this case, for low  $p$ , the incentive to invest in inventory is high, and cash holdings are less important because of the high opportunity costs of savings. For high  $p$ , savings become a relatively more effective means of transferring net worth to futures states, as inventory becomes too expensive. Hence, cash holdings have more value. When productivity is low, the incentive to invest and the realized net worth are low. In this case, saving are more valuable for low  $p$ —given the persistence of commodity price, this is a state which will likely be followed by another state with low  $p'$ . In this state, cash holdings are more apt to transfer net worth to the future than inventory, because they are noncontingent. The opposite is true when  $p$  is high.

Next, to explain how inventory and cash policies are determined by cash flow volatility conditional on financing constraints, we show in which circumstances cash holdings provide the greatest benefit when combined with inventory management. We conduct comparative statics on the optimal inventory and cash holdings with respect to cash flow risk. In Figure 6, we compare the unconditional average inventory ( $n/k$ ) and cash ( $c/k$ ) calculated from a simulated economy for different values of  $\sigma_z$  (Panels A and B),  $\sigma_p$  (Panels C and D), and  $\rho$  (Panels E and F). All the other parameters are as in Table I. As before, we compensate for the variation in the unconditional average of cash flow,  $\pi(s)k^\alpha - \psi$ , due to the change in the relevant parameter. We scale inventory and cash holdings by capital in order to isolate changes in the ratios that are only due to changes in the numerator as much as possible. For comparison, in the figure, we also show the inventory ratio computed from a simulation of the model with only inventory (‘no cash’) and the cash ratio from a simulation with only cash holdings (‘no inventory’).

In Figure 6, Panel A, average inventory is increasing in  $\sigma_z$ : as net worth risk increases, the probability of equity issuance increases and the firm demands more risk management. Also, average cash holdings increase with  $\sigma_z$ , but this effect is visibly stronger when savings complement the operational hedge using inventory. Remarkably, Panel A shows that, with savings, inventory is higher for all  $\sigma_z$ . Furthermore, average cash holdings are significantly increased for any level of  $\sigma_z$  when the firm is allowed to manage inventory, as shown in Panel B. Therefore, the analysis in Panels A and B is consistent with the synergic role of cash described in Section II.D.

The effect of an increase in  $\sigma_p$  on inventory and cash holdings is stronger than the one of  $\sigma_z$ , and, as expected, average  $n/k$  is always monotonically increasing in  $\sigma_p$  in Panel C. Cash holdings complement risk management using storage, as the inventory ratio is higher when the firm is allowed to save. For very low or very high  $\sigma_p$ , average inventory is approximately the same with or without savings. The model predicts a strong complementarity between inventory and savings due to financing constraints and net worth risk driven by commodity price risk. This is clear in Panel D: average cash holdings are monotonically increasing in commodity price risk. The higher sensitivity of cash holdings to  $\sigma_p$  is a reflection only of the sensitivity of inventory to commodity price risk, given the higher need of financing the inventory investment using internal resources if  $\sigma_p$  is high. Indeed, when the firm cannot manage inventory, average cash holdings are almost insensitive to commodity price risk.

In Panel E of Figure 6, inventory increases with  $\rho$ . This result may at first seem counterintuitive because a higher correlation reduces risk and should lead to less risk management. However, a high correlation increases the likelihood that the cash flow is high enough to finance inventory investment. For example, the average of  $n/k$  conditional on  $p < 1$  (i.e., when investment in inventory is more likely), is 0.43 for  $\rho = 0$  and 0.62 for  $\rho = 1$ . In Panel F, as seen above, savings are more valuable with than without inventory and average cash holdings are decreasing in  $\rho$  for a firm with inventory, as internal financing of investments is more likely when  $\rho$  is high.

In summary, the calibrated model shows that savings complement inventory in risk management. Also, a shortage of internal funds is more likely when productivity risk is high and correlation is low; therefore, savings are important for sustaining inventory in those cases.

## D.2. Empirical evidence on the complementarity of savings and inventory

We contrast these predictions on the interaction between cash holdings and inventory to empirical data by reporting the actual use of inventory and cash holdings for financially constrained and unconstrained firms with different levels of cash flow volatility. Table VII shows the holdings of inventory and cash (scaled by capital) in our Compustat sample of manufacturing firms, which we obtained by double-sorting firms on the tightness of financing constraints and cash flow volatility.

By sorting with respect to the latter variable, we capture the effects described in the model: higher volatility and lower correlation of the two shocks generate higher unconditional cash flow risk.

Specifically, we compute firm’s cash flow volatility as the standard deviation of cash flow (ebit in Compustat) scaled by total assets over the entire time span in which the firm is observed. Also, we compute the tightness of financing constraints using three alternative proxies: size (natural log of total assets), a dividend payment dummy, and the Whited and Wu (2006) (WW) index.<sup>11</sup> We classify firms in each year as financially constrained (unconstrained) if they belong to the first (fourth) quartile of the size distribution, they do not pay (do pay) dividends, and their WW index is in the fourth (first) quartile of the WW index distribution. A popular proxy of financing constraints is the Kaplan and Zingales (1997) index. However, this index cannot be used in our setup because it is computed using the cash ratio, which is endogenous in our analysis.

In Table VII, financially constrained firms hold more inventory and cash than their financially unconstrained counterparts, regardless of the proxy used to measure financing constraints. For both inventory and cash holdings, we test the null hypothesis of equal means between groups of firms with an equal degree of financing constraints but different cash flow volatilities, and between groups with equal cash flow volatility but different tightness of financing constraints. We reject the null at the 1% level in each test, except in the one testing equal means of inventory between groups with different cash flow volatility classified as constrained using the WW index, where the null hypothesis is rejected at the 5% level.

White, Pearson, and Wilson (1999) found that smaller firms typically rely more on inventory than larger firms, as larger firms can afford more efficient (e.g., just-in-time) production systems. However, Chen, Frank, and Wu (2005) found that, although a more efficient supply chain undoubtedly reduces the need to hold an inventory of raw materials, manufacturing firms prefer to store raw materials to cope with risks not eliminated by advanced supply chain technologies. One such risk is given by prices fluctuations, as remarked by Chen, Frank, and Wu (2005), who found a signif-

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<sup>11</sup>The index of financing constraints estimated by Whited and Wu (2006) is

$$WW_{it} = -0.091CF_{it} - 0.062DIVPOS_{it} + 0.021TLTD_{it} - 0.044LNTA_{it} + 0.102ISG_{it} - 0.035SG_{it},$$

which is a linear combination of: the ratio of long-term debt to total assets (TLTD), the dividend indicator (DIVPOS), size (LNTA), the ratio of cash flow over total assets (CF), the firm’s sales growth (SG), and the firm’s three-digit industry sales growth (ISG). The index can take either sign and directly measures financial constraints (i.e., the more financially constrained a firm is, the higher the WW index).

icant positive relation between inflation and raw materials holdings, showing that manufacturing firms are actually sensitive to price risk. Our analysis in Section III.C on inventory investment complements their evidence.

More importantly, we find that net worth risk and financing constraints positively impact the incidence of *both* inventory and cash holdings in the cross-section of manufacturing firms. This result can only be rationalized by the complementarity between inventory and cash, as predicted by our model. This marks an important distinction with respect to the previous literature on inventory and financing constraints (e.g. Dasgupta, Li, and Yan 2016), which more or less implicitly assumes near perfect substitutability between cash holdings and inventory, as the latter is seen as a costly reserve of liquidity. However, there are circumstances in which inventory is superior, given its contingent nature, and therefore the manager prefers it to cash holdings. It is exactly because of the high incentive to use inventory in some future states that the savings are important in the presence of real frictions as a means to transfer net worth to future states in which the commodity price is low.

For robustness, we computed ratios sorting firms with respect to the volatility of cash flow, where cash flow is computed as earnings plus depreciation ( $ib + dp$  in Compustat) or as operating income ( $oibdp$ ) instead of  $ebit$ , and found equivalent results. Furthermore, we computed ratios for firms in the lowest and highest quartiles of the cash flow volatility distribution. We find similar results to those reported in Table VII for both inventory and cash holdings. The only exception is that cash ratios are increasing in size and decreasing in the WW index for firms in the highest quartile of cash flow volatility. For these firms, cash holdings are so high that the effect of agency issues, which we do not control for, cannot be excluded. On the contrary, input inventory cannot be easily diverted, as argued by Burkart and Ellingsen (2004). Therefore, inventory holdings are much less affected by managerial agency issues.

### **D.3. Inventory, cash holdings, and capital investment**

Risk management is relevant in relation to the prospect of financing capital investment. It is then natural to further investigate the described complementarity of inventory and cash holdings in the cross-section, for low vs high investment firms. In Table VIII, we report average inventory and cash

ratios (scaled by capital) computed on simulated and empirical firms, sorted relative to the median capital investment rate. In the model, besides the base case, we consider also a low ( $\sigma_z = 0.10$ ) vs high ( $\sigma_z = 0.30$ ) cash flow volatility case. Similarly, for empirical data, in addition to the full sample, we show sorts based on the median cash flow volatility.

Simulated data show that both inventory and cash holdings are increasing in capital investment rate for any level of cash volatility considered. A similar pattern is visible in the empirical sample, although with a higher sensitivity of cash holdings to cash flow volatility. Overall, the model helps rationalize the empirical results: integrated risk management using inventory and savings is more significant when risk, driven by cash flow volatility, is higher and financing needs, driven by investment, are bigger.

To illustrate how inventory and cash holdings support capital investment, in Table IX we regress investment on its first lag, and on the market-to-book ratio, cash flow, current inventory, and current cash holdings. We do this in the model and in the data. Not surprisingly, investment is persistent and positively correlated with market-to-book ratio and with cash flow (although the cash flow sensitivity become insignificant with the EJW estimator for the reasons we explained above). More importantly, both inventory and cash holdings are positively related to capital investment, and this is true in the model and in the data. However, when we run the regression on standardized variables, investment is twice as sensitive to current inventory than it is to current cash holdings, and this occurs both in the model and in the data. This result is consistent with the idea that inventory has a more direct and prominent role than savings in risk management.

The model shows that savings complement the operational hedge by increasing inventory investment in future states. Such complementarity can essentially be seen in the time-series dimension. In Table X we regress, using simulated and empirical data, inventory on cash holdings using a dynamic specification that includes lags of both variables.<sup>12</sup> In Panel A, we observe that the regression coefficient related to contemporaneous cash holdings is negative in each column, suggesting a contemporaneous substitution between inventory and cash holdings due to the budget constraint.

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<sup>12</sup>In terms of explanatory power, the relevant variable for inventory holdings is its first lag, while the lags of both inventory and cash holdings are useful to reduce the autocorrelation of residuals. Because residuals are significantly correlated up to the first lag, we estimate standard errors robust to residuals autocorrelation. We also estimated the same regression adding more lags of both the inventory and cash ratios to completely clear autocorrelation and found similar results. We do not include additional regressors (market-to-book, size, cash flow, etc.), because they cause no sensible variation in the  $R^2$ .

Secondly, the coefficient related to the first lag of the cash ratio is positive. To gauge the intertemporal relation between inventory and savings, one has to sum all the coefficients related to the latter variable. In our case, this gives -0.23, which is lower than -0.69 when only the contemporaneous cash ratio is used as explanatory variable. This result confirms that, in the model, a firm with more savings is better positioned to take advantage of the operational hedge offered by inventory. This intuition is confirmed in the data, Panel B. The estimated coefficient of contemporaneous cash ratio is always negative and significant. More importantly, there is evidence of a positive effect of lagged cash ratio on inventory, supporting the idea of a dynamic complementarity between inventory and cash holdings.

Finally, because we cannot generally observe the incidence of derivatives usage in our empirical sample, we abstract from this when studying the evidence on the complementarity between savings and inventory. If this omission introduces a bias, it is actually in favor of our results: firms that can also use derivatives to hedge against input price risk have less inventory, everything else equal, and the complementarity between inventory and savings that we find empirically is still strong.

## IV. Conclusion

We have examined the contribution of inventory to corporate risk management in the context of a model in which the firm dynamically invests in capital, manages commodity price risk using storage, and saves in the presence of costly external finance and endogenous default.

In our model, risk management using inventory is implemented by firms with different levels of net worth. Remarkably, we found that firms with low net worth should still engage in risk management. This is because inventory does not require collateral or margins, which would impose a trade-off between external financing and risk management for a low level of net worth, and because the cost of risk management using inventory is determined by the current price of the commodity. The model predicts that there should be a positive and significant sensitivity of inventory investment to expected price change regardless of the level of net worth. This prediction is confirmed in the empirical data using a Compustat sample of manufacturing firms.

As pointed out by Froot, Scharfstein, and Stein (1993), risk management helps firms to smooth investment in capital when the production technology has decreasing returns to scale and it is costly to raise external finance. In the model, we found that the ability to manage net worth risk using inventory allows the firm to increase investment and the value of the firm, even when the main source of risk is not commodity price.

Inventory and cash holdings are typically considered substitutes for each other in operations and in generating liquidity. The intensity of the substitution between inventory and cash is enhanced by technological and regulatory innovations taking place over time. We have shown that inventory and cash holdings can also be complementary risk management tools. Such complementarity hinges on the fact that inventory of the commodity used in production is a natural hedge, because it is negatively correlated with the cash flow from production (in other words, inventory value is high when production is low). Therefore, firms with high net worth risk have a high incentive to use inventory to transfer net worth to future states. However, because of this contingent nature, inventory will be a poor reserve of net worth in future states in which the price is low, which is exactly when the incentive to invest in inventory will be higher. To transfer net worth to these states, cash holdings are more effective because they are non-contingent. In the model, the intensity of the synergy between inventory and savings is crucially determined by cash flow volatility, financing constraints, and capital adjustment costs.

We found support for this prediction in empirical data on firms in the manufacturing industry, for which net worth risk and financing constraints increase the incidence of both inventory and cash holdings together. Also, we have documented that the complementarity between the two risk management tools is enhanced when a firm has a higher investment rate and higher risk. Finally, we found empirical support for the predictions that investment in capital stock is more sensitive to lag inventory than to lag savings, and that savings are used to enhance the operational hedge implemented by inventory, in that lag savings are on average positively related to inventory investment.

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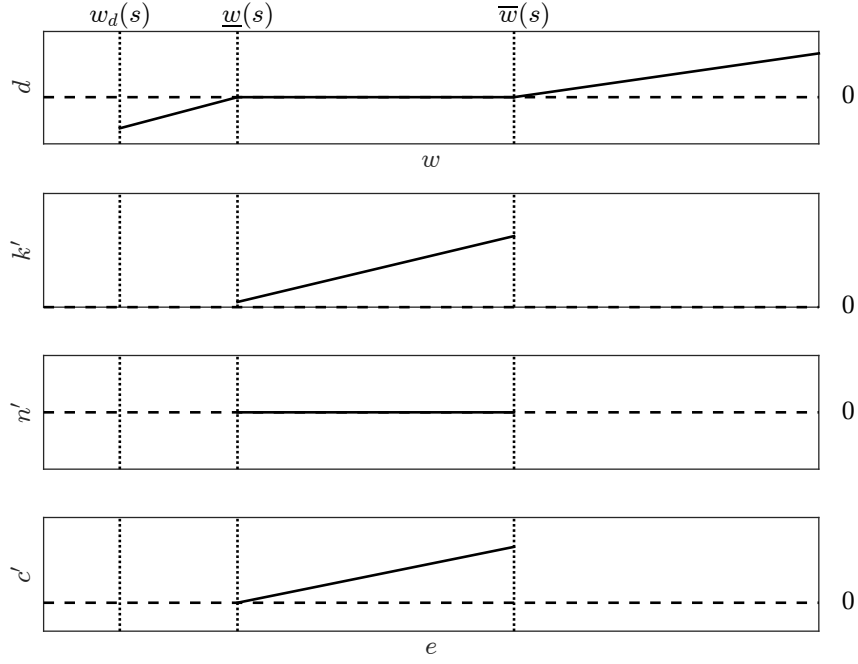
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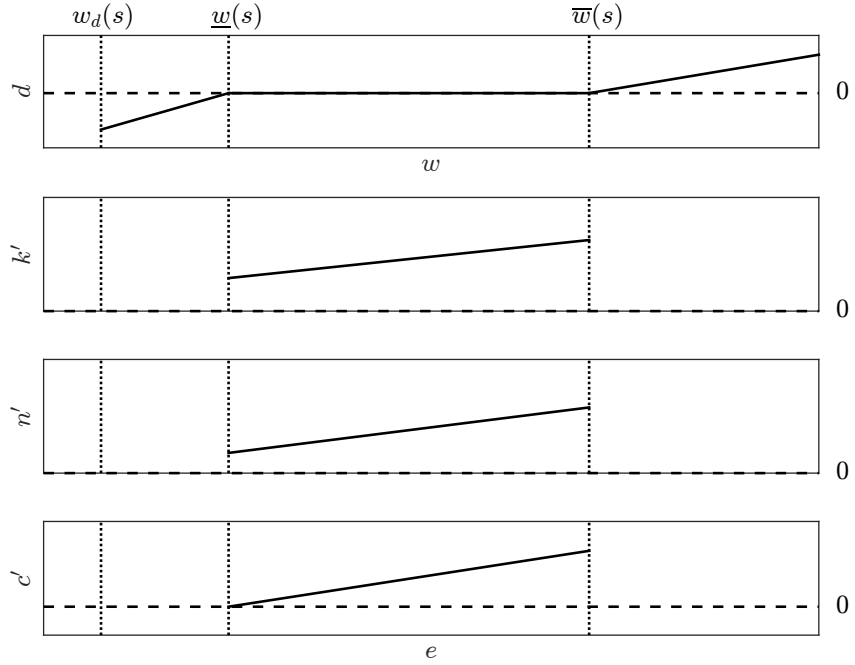
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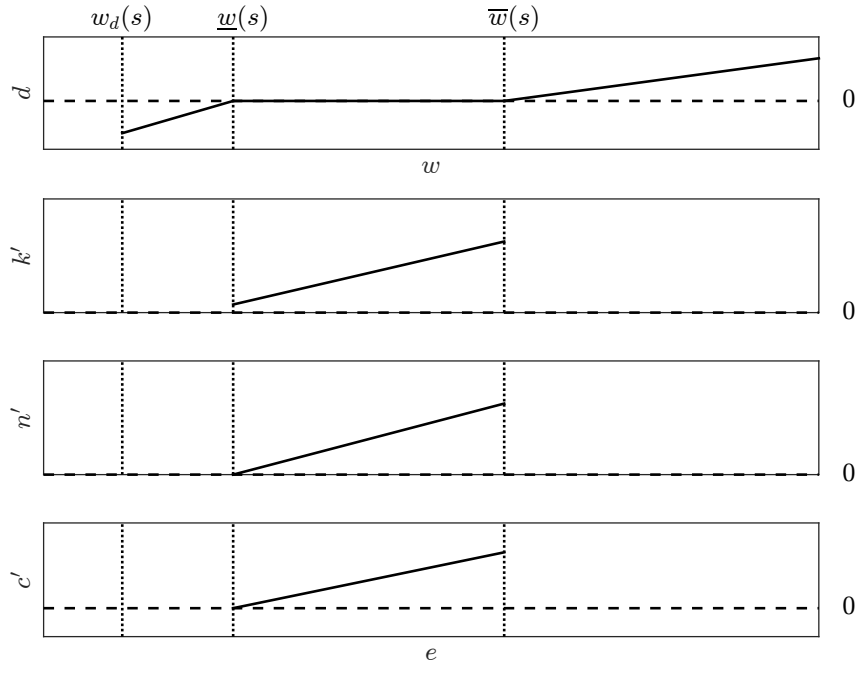


(A) High  $p$ , average  $z$

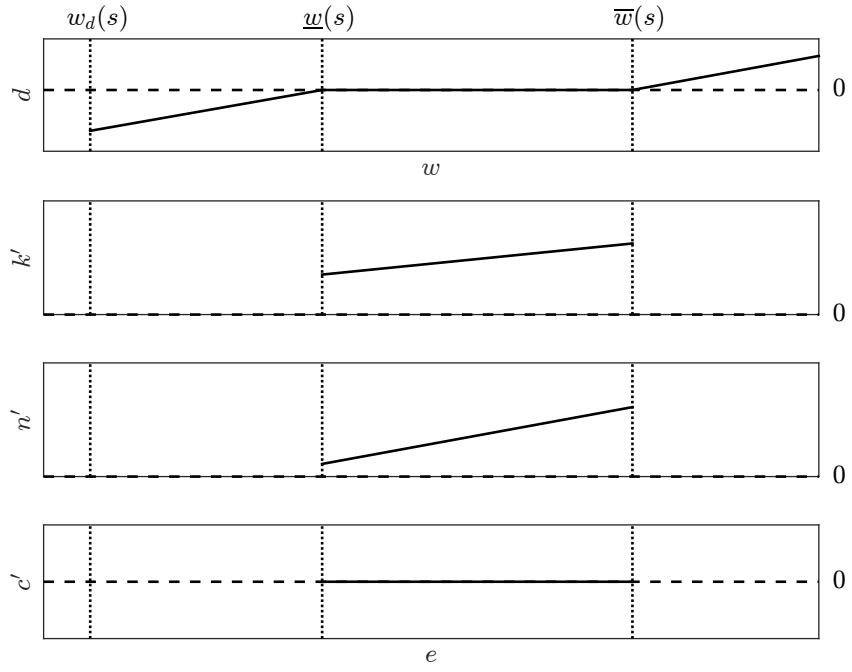


(B) Low  $p$ , average  $z$

Figure 1: **Optimal policies.** We plot optimal  $d$ ,  $k'$ ,  $n'$ , and  $c'$  against net worth  $w$ , at four possible states,  $s = (z, p)$ . In this figure,  $w_d(s)$ ,  $\underline{w}(s)$ , and  $\overline{w}(s)$  are as defined in Proposition 2. The dividend policy,  $d$ , is plotted against the realized net worth,  $w$ . Once the adjusted net worth,  $e = w - d$ , is determined, the policies for  $k'$ ,  $n'$ , and  $c'$  are plotted against  $e$ . We set parameters to values reported in Table I.



(C) Low  $p$ , low  $z$



(D) Low  $p$ , high  $z$

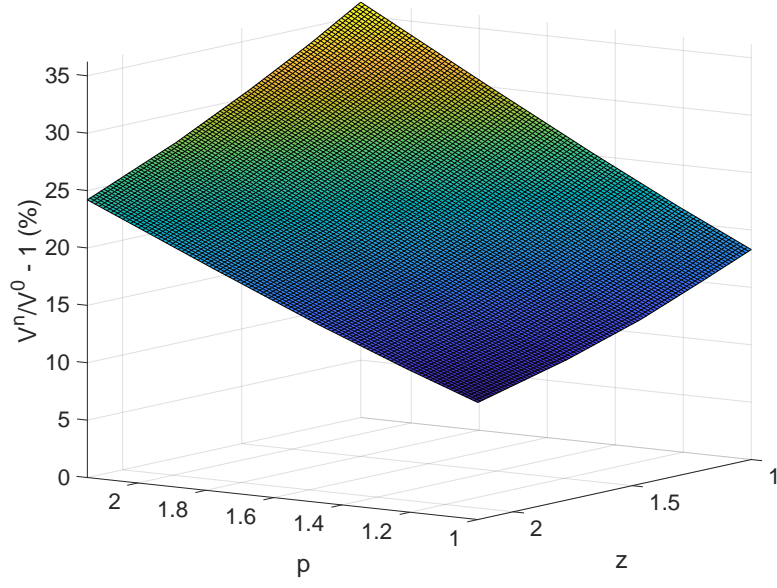


Figure 2: **The value of inventory management.** We plot against  $z$  and  $p$  the value created by inventory as a tool to manage net worth risk risk. The value is computed as  $V^n/V^0 - 1$ , where  $V^n$  is the value with inventory and no cash holdings, and  $V^0$  is the value in the model with no inventory and no cash holdings. The plot is based on  $k$  and  $n$  equal to their unconditional averages calculated from a simulated sample using the methodology described in Online Appendix B and the parameters in Table I.



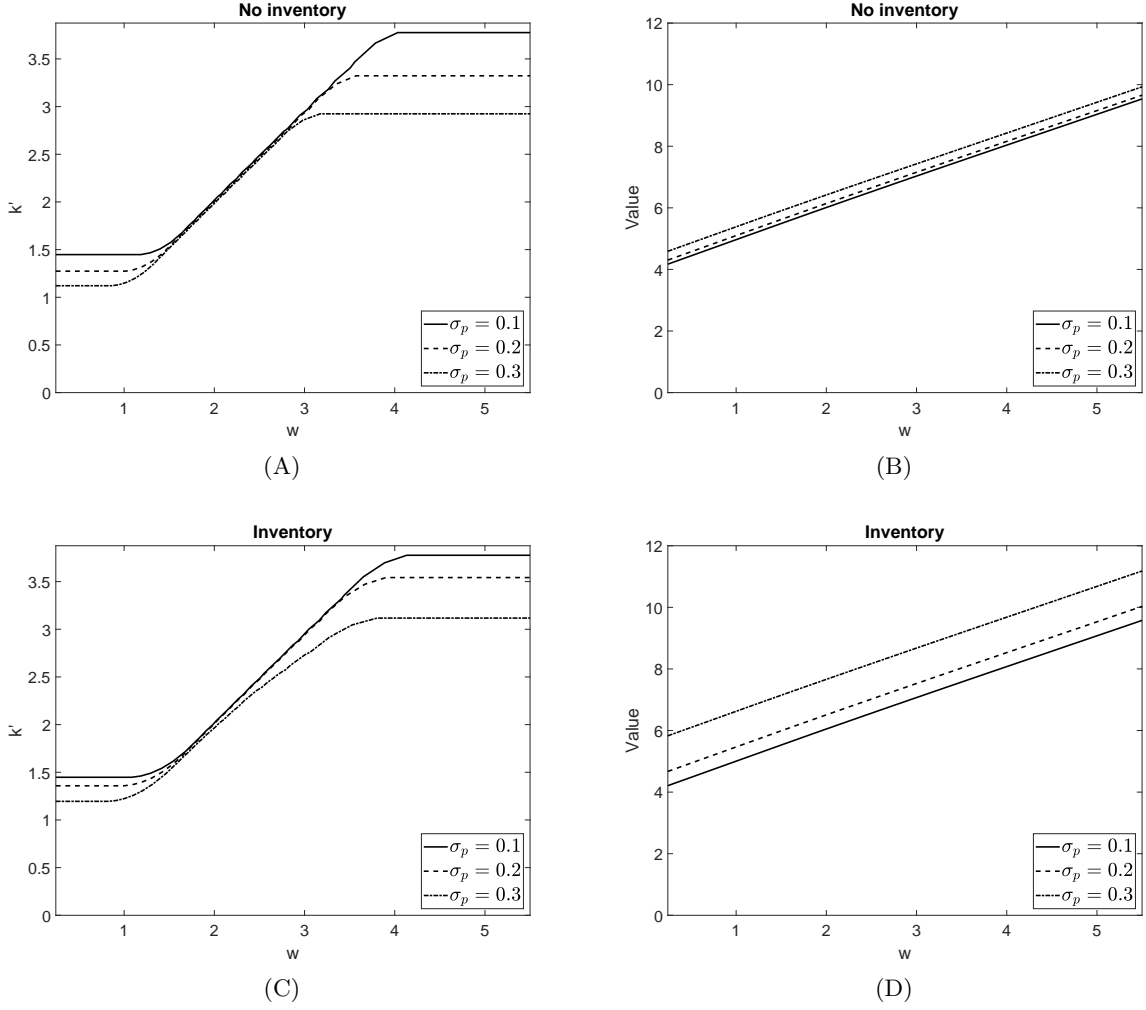


Figure 3: **Commodity price risk, investment, and firm value.** We plot against net worth  $w$  the optimal  $k'$  and firm value in the model with no risk management (Panels A and B) and in the model with inventory management (Panels C and D). These plots are obtained with  $z$  and  $p$  at the unconditional averages  $z = 1$  and  $p = 1$ , for different values of the volatility of  $p$ . The unconditional average cash flow  $\pi(s)k^\alpha - \psi$  is kept constant across the different values of  $\sigma_p$  by adjusting the unconditional average price. All the other parameters are as in Table I.

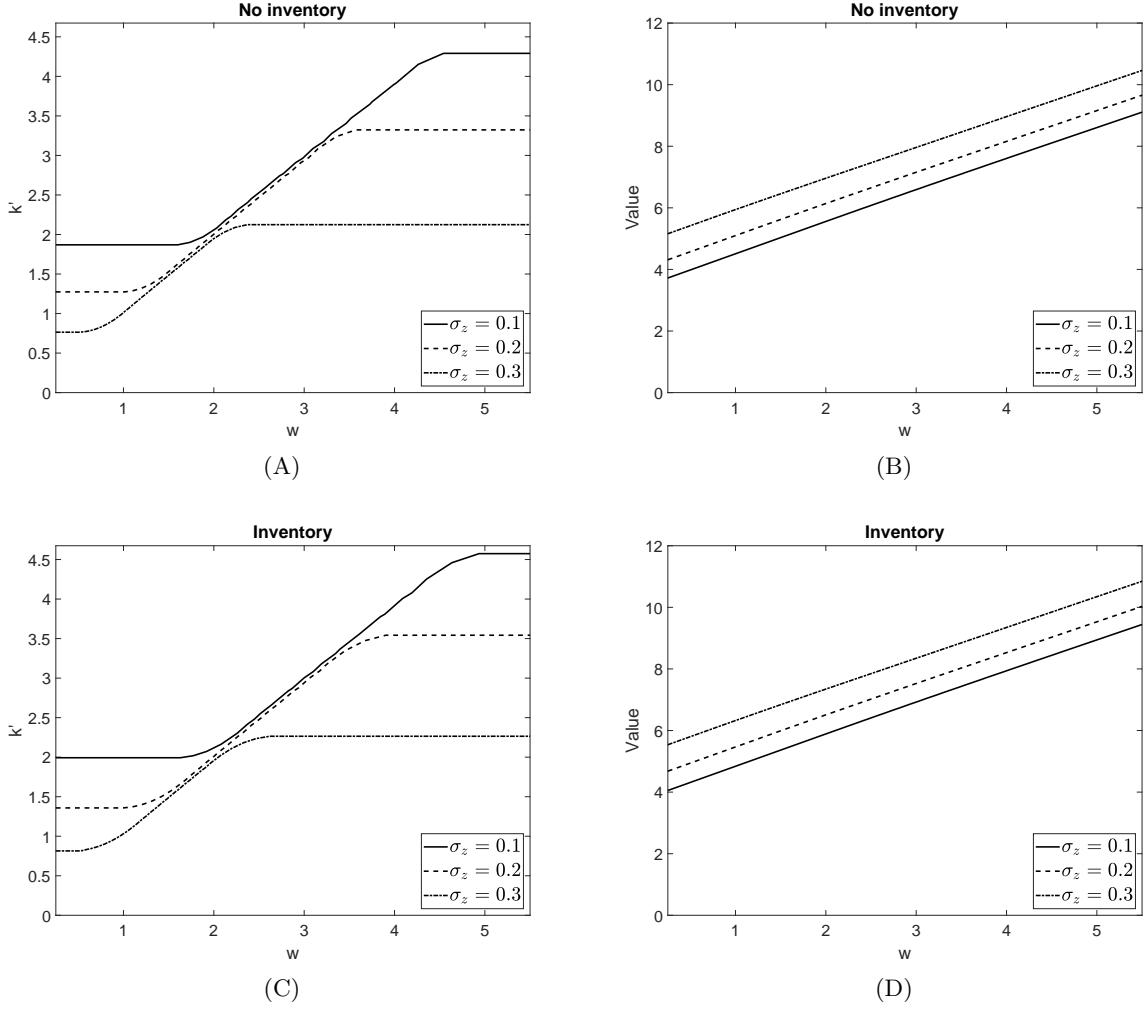
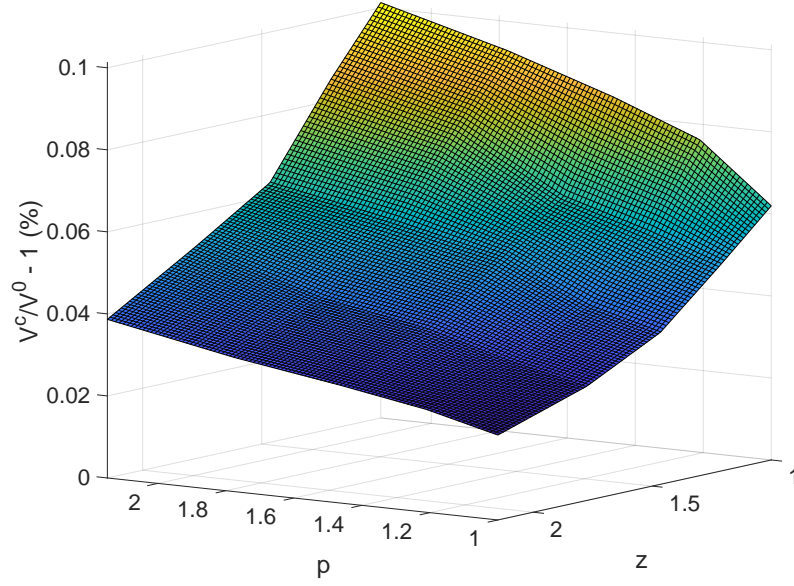
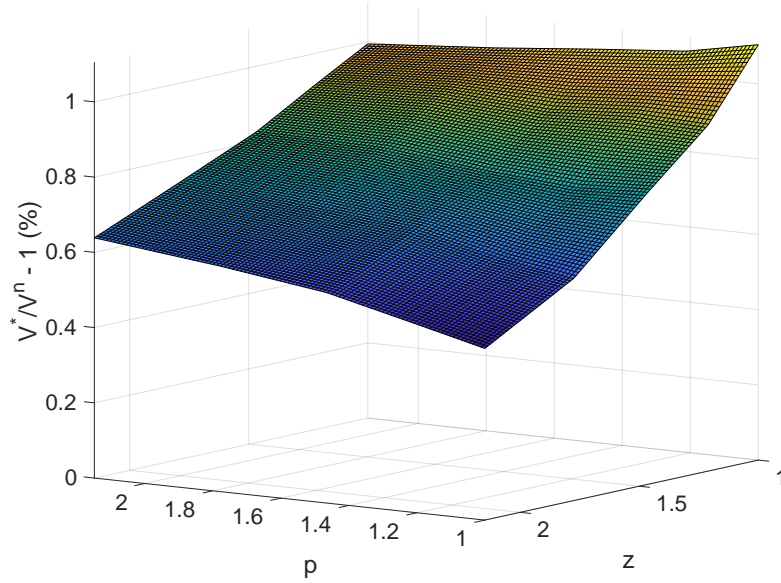


Figure 4: **Productivity risk, investment, and firm value.** We plot against net worth  $w$  the optimal  $k'$  and firm value in the model with no risk management (Panels A and B) and in the model with inventory management (Panels C and D). These plots are obtained with  $z$  and  $p$  at the unconditional averages  $z = 1$  and  $p = 1$ , for different values of the volatility of  $z$ . The unconditional average cash flow  $\pi(s)k^\alpha - \psi$  is kept constant across the different values of  $\sigma_z$  by adjusting the unconditional average price. All the other parameters are as in Table I.



(A)



(B)

Figure 5: **Interaction between inventory and cash holdings.** In Panel A, we plot against  $z$  and  $p$  the value created by cash holdings with no interaction with inventory, computed as  $V^c/V^0 - 1$ , where  $V^c$  is the enterprise value (i.e., after subtracting current cash holdings) in the model with dynamic management of cash holdings and no inventory, and  $V^0$  is firm value in the model with no inventory and no cash holdings. In Panel B, we plot the value created by cash holdings in the interaction with inventory, given by  $V^*/V^n - 1$ , where  $V^* = V - c$  is the enterprise value in the baseline model and  $V^n$  is the value with inventory but no cash holdings. The plot is based on  $k$ ,  $n$ , and  $c$  equal to their unconditional averages calculated from a simulated sample using the methodology described in Online Appendix B and the parameters in Table I.

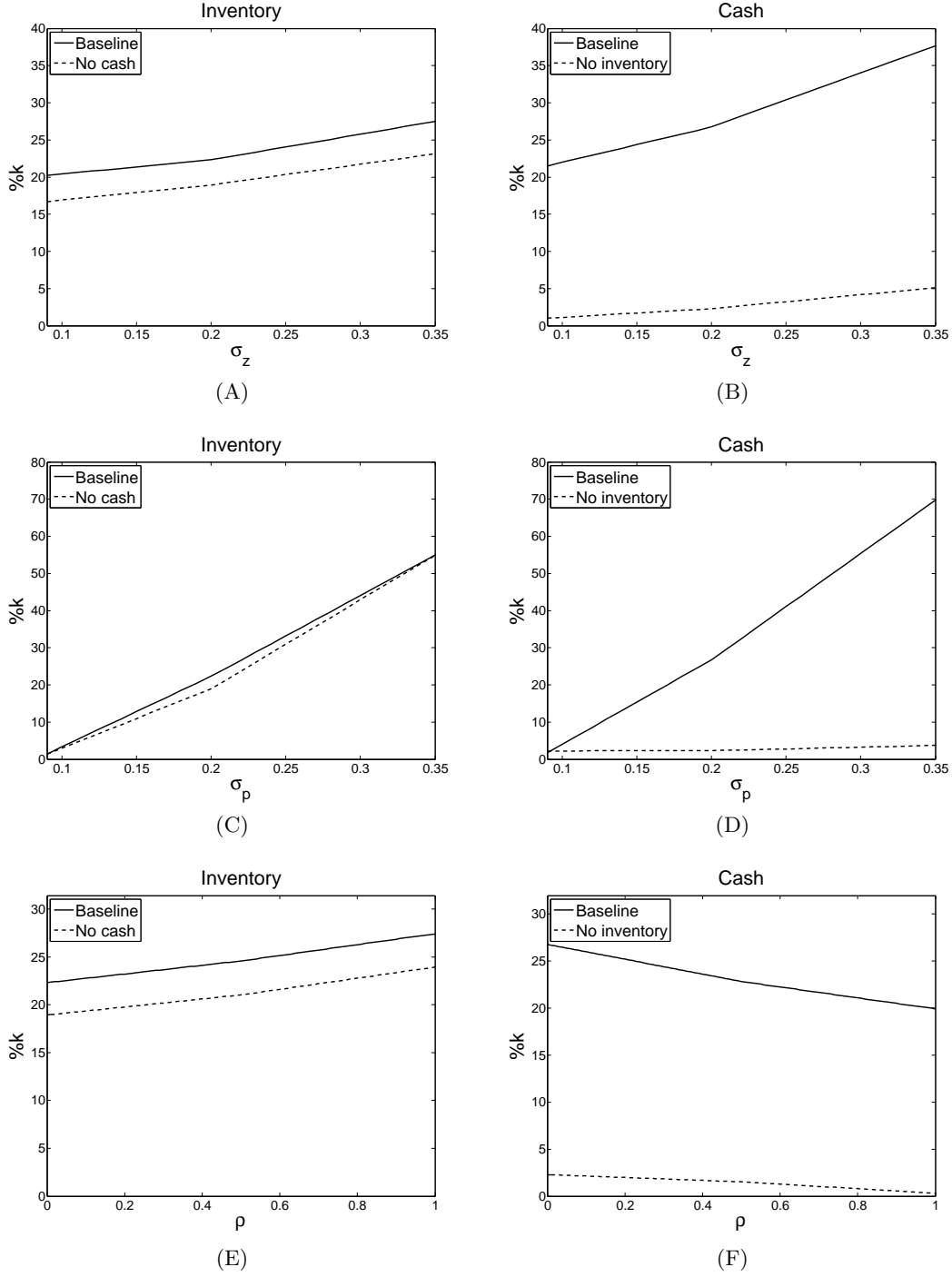


Figure 6: **Cash flow risk, inventory, and cash holdings.** We plot the unconditional average of inventory,  $n/k$ , and cash holdings,  $c/k$ , computed from simulated economies of the model. In Panels A and B we show a sensitivity with respect to  $\sigma_z$ , in Panels C and D a sensitivity to  $\sigma_p$ , and in Panels E and F a sensitivity to  $\rho$ . We also show  $n/k$  for the model with dynamic inventory but no cash holdings (‘no cash’), and  $c/k$  for the model with dynamic cash holdings but no inventory (‘no inventory’). The relevant parameter is varied while keeping the unconditional average cash flow  $\pi(s)k^\alpha - \psi$  constant by adjusting the average  $z$  or  $p$ . In all cases, all the other parameters are as in Table I. The numerical methods are described in Online Appendix B.

Parameter	Symbol	Value
Capital productivity	$\theta$	0.36
Commodity productivity	$\gamma$	0.54
Capital depreciation rate	$\delta$	0.12
Inventory storage costs	$\eta$	0.034
Capital adjustment costs	$\bar{\xi}/\underline{\xi}$	0.75/7.5
Fixed operating costs	$\psi$	0.03
Autoregression of log of $z_t$	$\phi_z$	0.62
Conditional standard deviation of log of $z_t$	$\sigma_z$	0.20
Autoregression of log of $p_t$	$\phi_p$	0.62
Conditional standard deviation of log of $p_t$	$\sigma_p$	0.20
Correlation between logs of $z_t$ and $p_t$	$\rho$	0.00
Equity issuance costs	$\lambda$	0.05
Discount factor	$\beta$	1/1.05
Return on cash holdings	$r$	0.0462

Table I: **Base case parameters.** Parameters from the calibration of the model to empirical data.

	Data	Model
Capital investment/capital, mean	0.13	0.13
Capital investment/capital, std.dev.	0.13	0.13
Probability of negative capital investment	0.03	0.06
Inventory investment/capital, mean	0.02	0.01
Inventory investment/capital, std.dev.	0.11	0.36
Probability of negative inventory investment	0.25	0.19
Inventory/assets, mean	0.08	0.12
Inventory/capital, mean	0.23	0.22
Inventory/capital, std.dev.	0.31	0.36
Cash/assets, mean	0.17	0.17
Cash/assets, std.dev.	0.21	0.15
Market/book, mean	1.84	1.84
Frequency of equity issuance	0.18	0.14

Table II: **Calibration.** This table presents the moments used to calibrate the model. Empirical data are a sample of firms in the Compustat North America database in the manufacturing industry (SIC codes 2000-3999) observed between 1969 and 2014. The investment rate in capital stock is calculated as capital expenditures (capx in Compustat) less sale of capital (sppe) scaled by beginning-of-period capital (ppeg). Inventory investment is the change of the stock of raw materials (invrm) between two consecutive years scaled by beginning-of-period capital. The inventory ratio is taken scaling inventory holdings scaled either by the beginning-of-period capital or total assets, and the cash ratio is cash and short-term securities (che) scaled by total assets (at). The market-to-book ratio is the sum of total assets and the market value of equity (prcc x csho) minus the book value of equity (ceq) and deferred taxes (txdb), scaled by total assets. Equity issuance is net sale of common and preferred stock (sstk - prstk) to which we apply a filter to consider only firm-initiated issuances. The moments from the model are based on a Monte Carlo simulation of the model. See Online Appendix B for details.

Inventory investment		Net worth			
		(Low)	(2)	(3)	(High)
Price change	1.28*** (0.01)	1.41*** (0.02)	1.31*** (0.01)	1.34*** (0.01)	1.20*** (0.01)
Current inventory	-0.86*** (0.00)	-0.77*** (0.02)	-0.80*** (0.01)	-0.86*** (0.00)	-0.93*** (0.00)
Market-to-book	0.09*** (0.00)	0.07*** (0.00)	0.14*** (0.01)	0.13*** (0.01)	0.20*** (0.00)
Cash flow	-0.20*** (0.00)	-0.19*** (0.02)	-0.44*** (0.03)	-0.39*** (0.02)	-0.43*** (0.01)
Low net worth	-0.04*** (0.01)				
Low net worth x price change	0.10*** (0.01)				
Adjusted-R <sup>2</sup>	0.690	0.470	0.580	0.770	0.880

Table III: **Sensitivity of inventory investment to expected commodity price changes.** We report estimates of a regression of investment in inventory,  $p_t i^n / (k_t + p_t n_t)$ , on the expected change of the commodity price,  $\mathbb{E}[p_{t+1}|p_t]/p_t - 1$ , in which  $\mathbb{E}[p_{t+1}|p_t] = p_t^{\phi_p} e^{\sigma_p^2/2}$ , on current inventory,  $p_t n_t / (k_t + p_t n_t)$ , on market-to-book,  $V_t / (k_t + p_t n_t)$ , and on cash flow,  $(\pi_t k_t^\alpha - \psi) / (k_t + p_t n_t)$ . Data are obtained by simulating the model, as described in Online Appendix B. OLS regression coefficients are estimated based on demeaned variables for the full sample, including a low-net-worth dummy for below median net worth and an interaction term between this dummy and price change (first column), and for net worth quartile groups of firms (remaining four columns). Bootstrapped standard errors are reported in parentheses. The reported coefficient and standard error of the low-net-worth dummy are given by the actual estimates multiplied by 10. \*\*\* indicate significance at the 1% level.

Inventory investment	Capex		Volatility	
	Low	High	Low	High
Price change	1.15*** (0.01)	1.39*** (0.01)	1.39*** (0.01)	1.25*** (0.01)
Current inventory	-0.89*** (0.00)	-0.81*** (0.00)	-0.88*** (0.00)	-0.85*** (0.00)
Market-to-book	0.07*** (0.00)	0.13*** (0.00)	0.10*** (0.00)	0.08*** (0.00)
Cash flow	-0.20*** (0.01)	-0.26*** (0.01)	-0.21*** (0.01)	-0.19*** (0.00)
Low net worth	-0.01*** (0.00)	-0.03*** (0.00)	-0.01*** (0.00)	-0.01*** (0.00)
Low net worth x price change	-0.10*** (0.02)	0.42*** (0.02)	-0.44*** (0.01)	0.42*** (0.02)
Adjusted-R <sup>2</sup>	0.70	0.72	0.74	0.66

Table IV: **Inventory investment and firm characteristics.** We report estimates of a regression of inventory investment,  $p_t i^n / (k_t + p_t n_t)$ , on the expected change of the commodity price,  $\mathbb{E}[p_{t+1}|p_t]/p_t - 1$ , in which  $\mathbb{E}[p_{t+1}|p_t] = p_t^{\phi_p} e^{\sigma_p^2/2}$ , on current inventory,  $p_t n_t / (k_t + p_t n_t)$ , on market-to-book,  $V_t / (k_t + p_t n_t)$ , and on cash flow,  $(\pi_t k_t^\alpha - \psi) / (k_t + p_t n_t)$ . We include a low-net-worth dummy for below median net worth, and an interaction term between this dummy and price change. Data are obtained by simulating the model, as described in Online Appendix B. We sort the sample based on the capital investment rate,  $i^k/k$ , for the base case. The sorts based on volatility are obtained by generating samples with low ( $\sigma_z = 0.10$ ) and high ( $\sigma_z = 0.30$ ) cash flow volatility. OLS regression coefficients are estimated based on demeaned variables. Bootstrapped standard errors are reported in parentheses. \*\*\* indicate significance at the 1% level.



Inventory investment		Net worth			
		(Low)	(2)	(3)	(High)
Price change	0.16*** (0.01)	0.06** (0.02)	0.05*** (0.02)	0.17*** (0.02)	0.17*** (0.04)
Current inventory	-0.21*** (0.01)	-0.30*** (0.02)	-0.35*** (0.02)	-0.29*** (0.02)	-0.01 (0.07)
Market-to-book	0.02*** (0.00)	0.03*** (0.01)	0.03*** (0.01)	0.03*** (0.01)	0.04** (0.02)
Cash flow	0.04*** (0.00)	0.08*** (0.01)	0.05*** (0.01)	-0.00 (0.02)	-0.13 (0.09)
Low net worth	0.01*** (0.00)				
Low net worth x price change	-0.09*** (0.02)				
Observations	66168	15974	17185	17033	15978
Adjusted-R <sup>2</sup>	0.18 (0.01)	0.22 (0.02)	0.23 (0.02)	0.21 (0.03)	0.12 (0.03)
$\tau^2$	0.14 (0.01)	0.13 (0.03)	0.15 (0.03)	0.17 (0.03)	0.18 (0.03)

Table V: **Inventory investment and expected price changes.** We estimate the sensitivity of investment in raw material inventory (invmr minus lagged invrm in Compustat) to expected price change conditional on the level of book net worth (lagged seq) for the sample described in Section III.A. The control variables are current inventory (lagged invrm), market-to-book ((at + prcc x csho - ceq - txdb)/at), and cash flow (ebit). All variables are scaled by the beginning-of-period total assets (lagged at), except expected price change, computed assuming an AR(1) specification and using the time series of the PPI retrieved from the FRED database. Estimates are based on variables demeaned at the firm-level, for the entire sample including a low-net-worth dummy for below median net worth and an interaction term between this dummy and price change (first column), and for net worth quartile groups of firms (remaining four columns). Regression coefficients are estimated using the Erickson, Jiang, and Whited (2014) third-order cumulant estimator.  $\tau^2 \in (0, 1)$  is an index of measurement quality related to market-to-book.  $\tau^2 = 1$  indicates perfect measurement. Standard errors robust to within firm serial correlation are reported in parentheses. \*, \*\*, and \*\*\* indicate significance at the 10%, 5%, and 1% levels respectively.



Inventory investment	Capex		Volatility	
	Low	High	Low	High
Price change	0.15*** (0.02)	0.17*** (0.02)	0.20*** (0.03)	0.11*** (0.02)
Current inventory	-0.27*** (0.02)	-0.20*** (0.02)	-0.14*** (0.03)	-0.24*** (0.01)
Market-to-book	0.03*** (0.01)	0.03*** (0.01)	0.03*** (0.01)	0.02*** (0.00)
Cash flow	0.05*** (0.01)	0.03*** (0.01)	0.04 (0.03)	0.04*** (0.00)
Low net worth	0.01*** (0.00)	0.01*** (0.00)	0.02*** (0.00)	0.01*** (0.00)
Low net worth x price change	-0.12*** (0.03)	-0.07** (0.04)	-0.13*** (0.02)	-0.04 (0.03)
Observations	27307	25204	33289	32879
Adjusted-R <sup>2</sup>	0.17 (0.02)	0.18 (0.02)	0.12 (0.01)	0.21 (0.01)
$\tau^2$	0.08 (0.03)	0.11 (0.02)	0.10 (0.02)	0.15 (0.02)

Table VI: **Inventory investment regression and firm characteristics.** We estimate the sensitivity of investment in raw material inventory (invmr minus lagged invrm in Compustat) to expected price change for the sample described in Section III.A. We sort the sample based on the capital investment rate ((capx - sppe)/ppegt in Compustat), and cash flow (ebit/at) volatility computed as the standard deviation of cash flow over the firm's life. The control variables are current inventory (lagged invrm), market-to-book ((at + prcc x csho - ceq - txdb)/at), cash flow (ebit), and a low-net-worth (based on book net worth-lagged seq) dummy for below median net worth and an interaction term between this dummy and price change. All variables are scaled by the beginning-of-period total assets (lagged at), except expected price change, computed assuming an AR(1) specification and using the time series of the PPI retrieved from the FRED database. Estimates are based on variables demeaned at the firm-level. Regression coefficients are estimated using the Erickson, Jiang, and Whited (2014) third-order cumulant estimator.  $\tau^2 \in (0, 1)$  is an index of measurement quality related to market-to-book.  $\tau^2 = 1$  indicates perfect measurement. Standard errors robust to within firm serial correlation are reported in parentheses. \*, \*\*, and \*\*\* indicate significance at the 10%, 5%, and 1% levels respectively.

		Size		
Volatility	Constrained		Unconstrained	
Low	Inventory	Cash	Inventory	Cash
	0.30	0.87	0.11	0.28
	(0.34)	(2.51)	(0.12)	(0.73)
	Obs = 5390		14594	
High	0.36	1.42	0.13	1.15
	(0.43)	(3.27)	(0.18)	(2.35)
	Obs = 15894		5215	
		Dividends		
	Constrained		Unconstrained	
Low	Inventory	Cash	Inventory	Cash
	0.23	0.97	0.16	0.27
	(0.28)	(2.59)	(0.17)	(0.89)
	Obs = 15096		25785	
High	0.30	1.82	0.23	0.81
	(0.38)	(3.63)	(0.29)	(2.19)
	Obs = 30586		12992	
		WW Index		
	Constrained		Unconstrained	
Low	Inventory	Cash	Inventory	Cash
	0.29	0.92	0.10	0.24
	(0.33)	(2.66)	(0.11)	(0.54)
	Obs = 4450		13412	
High	0.33	1.40	0.13	0.83
	(0.40)	(3.09)	(0.17)	(1.92)
	Obs = 15385		3982	

Table VII: **Inventory and cash holdings.** We report average inventory of raw materials and the average cash holdings, both scaled by capital (ppeg in Compustat), for the sample of manufacturing firms from Compustat described in Section III.D. Firms are sorted into two subsets divided by the median of cash flow volatility, computed as the historical standard deviation of the firm's cash flow (ebit in Compustat) scaled by total assets for each firm, and on the tightness of financing constraints. Three measures are used for this purpose: size (the natural log of total assets), dividends (indicator variable equal to one if the firm pays cash dividends), and the Whited-Wu (WW) index. A firm is classified as financially constrained (unconstrained) if it belongs to the first (fourth) quartile of the size distribution, or it does not pay (does pay) dividends, or it belongs to the fourth (first) quartile of the WW index distribution. Standard deviations are in parentheses. The null hypothesis of equal means for the same variable across subgroups is tested using a standard t-test. The null hypothesis for all pairs of groups is rejected at the 1% level, except in the test comparing the means of inventory for constrained firms in the WW column with different cash flow volatility, for which the null hypothesis is rejected at the 5% level.

Panel A: Model				
Capex	Low		High	
	Inventory	Cash	Inventory	Cash
Base case	0.19 (0.32)	0.25 (0.26)	0.26 (0.39)	0.27 (0.28)
Low volatility	0.17 (0.28)	0.20 (0.17)	0.24 (0.34)	0.24 (0.21)
High volatility	0.22 (0.41)	0.33 (0.36)	0.30 (0.48)	0.35 (0.40)
Panel B: Data				
Capex	Low		High	
	Inventory	Cash	Inventory	Cash
Full sample	0.20 (0.26) Obs = 33662	0.83 (2.42)	0.28 (0.36) 33508	1.63 (3.53)
Low volatility	0.17 (0.20) Obs = 16470	0.41 (1.47)	0.21 (0.26) 13872	0.83 (2.34)
High volatility	0.23 (0.31) Obs = 16756	1.18 (2.88)	0.32 (0.41) 18710	2.08 (3.91)

Table VIII: **Inventory, cash holdings, and investment.** In Panel A, we report average inventory  $n_t/k_t$  and cash holding  $c_t/k_t$  for different values of the cash flow volatility and sorting on the capital investment rate  $i_t^k/k_t$  based on the median, using samples simulated as described in Online Appendix B. The sorts based on volatility are obtained by generating samples with low ( $\sigma_z = 0.10$ ) and high ( $\sigma_z = 0.30$ ) cash flow volatility. In Panel B, based on the empirical sample described in Section III.A, we report average inventory (invm in Compustat) and cash holding (che) scaled by capital (ppeg) sorting based on median capital investment rate  $((\text{capx} - \text{sppe})/\text{ppeg})$ . The sorts on volatility are based on the standard deviation of cash flow (ebit/at) over the entire time span in which the firm is observed. Differences of average values are all significant at the 1%.

	Panel A: Model		Panel B: Data	
	OLS	std OLS	EJW	std OLS
Capital investment				
Capital investment (t - 1)	0.09*** (0.00)	0.12*** (0.00)	0.27*** (0.01)	0.34*** (0.00)
Market-to-book	0.09*** (0.00)	0.21*** (0.00)	0.03*** (0.00)	0.01*** (0.00)
Cash flow	0.01*** (0.00)	0.13*** (0.00)	0.00 (0.01)	0.06*** (0.00)
Current inventory	0.03*** (0.00)	0.06*** (0.00)	0.07*** (0.01)	0.10*** (0.01)
Current cash holdings	0.01*** (0.00)	0.03*** (0.00)	0.03*** (0.00)	0.05*** (0.00)
Observations			55904	55904
Adjusted-R <sup>2</sup>	0.78	0.70	0.31 (0.01)	0.17
$\tau^2$			0.30 (0.02)	

Table IX: **Capital investment regression.** The table presents investment regressions in the model and in the data. As for the model, from the simulated economy described in Online Appendix B, we regress capital investment,  $i_t^k/(k_t + p_t n_t + c_t)$ , on its first lag and on the market-to-book ratio,  $V_t/(k_t + p_t n_t + c_t)$ , cash flow,  $(\pi_t k_t^\alpha - \psi)/(k_t + p_t n_t + c_t)$ , current inventory,  $p_t n_t/(k_t + p_t n_t + c_t)$ , and current cash holdings,  $c_t/(k_t + p_t n_t + c_t)$ . Bootstrapped standard errors are reported in parentheses. As for the sample described in Section III.A, we regress capital investment (capx minus sppe in Compustat), on its first lag, and on market-to-book ratio ((at + prcc x csho - ceq - txdb)/at), cash flow (ebit), current inventory (lagged invrm), and current cash holdings (lagged che). All variables are scaled by the beginning-of-period total assets (lagged at). Regression coefficients are estimated using OLS and OLS on standardized variables (std OLS) for simulated data, the Erickson, Jiang, and Whited (2014) third-order cumulant estimator (EJW) and OLS on standardized variables for empirical data. In the empirical regressions, we subtract from variables firm and year means.  $\tau^2 \in (0, 1)$  is an index of measurement quality related to market-to-book, with  $\tau^2 = 1$  indicating perfect measurement. Standard errors robust to within firm serial correlation are reported in parentheses. \*, \*\*, and \*\*\* indicate significance at the 10%, 5%, and 1% levels.

	Panel A: Model			Panel B: Data		
Inventory	(1)	(2)	(3)	(1)	(2)	(3)
Inventory (t - 1)		0.31*** (0.00)	0.51*** (0.00)		0.64*** (0.01)	0.67*** (0.01)
Cash holdings	-0.69*** (0.00)	-0.54*** (0.00)	-1.31*** (0.01)	-0.09*** (0.00)	-0.05*** (0.00)	-0.08*** (0.00)
Cash holdings (t - 1)			1.08*** (0.01)			0.05*** (0.00)
Adjusted-R <sup>2</sup>	0.25	0.34	0.53	86017 0.07	75943 0.47	75943 0.49

Table X: **Inventory and cash holdings regression.** In Panel A, we report estimates of a regression of inventory,  $p_t n_t$ , on cash holdings,  $c_t$ , including first lags, using total assets,  $k_t + p_t n_t + c_t$ , as scaling variable. Data are obtained by simulating the model, as described in Online Appendix B. OLS regression coefficients are estimated based on demeaned variables. Bootstrapped standard errors are reported in parentheses. In Panel B, for the sample described in Section III.A, we regress inventory holdings (invm in Compustat) on cash holdings (che), both scaled by total assets (at), including first lags. To control for unobserved heterogeneity, regression coefficients are estimated based on variables demeaned at the firm-level and including year dummies. To facilitate the comparability of results between the two panels, we exclude the intercept from the regression on simulated data. Standard errors robust to within firm serial correlation are reported in parentheses. \*, \*\*, and \*\*\* indicate significance at the 10%, 5%, and 1% levels.

## Online appendix

### A. Proof of propositions

Although the manager's decisions on dividends, investment in capital stock, inventory, and cash holdings occur on the same date  $t$ , we can separate them into two stages, as in Cooley and Quadrini (2001): in the first stage, a default/dividend decision is made; in the second stage the firm decides what investment to make in capital and inventory and how much to save.

The two-stage model of the firm is as follows. Proceeding backwards, the *ex dividend* value of equity is defined first as

$$\begin{aligned} v(s, e) &= \max_{x'} \beta \int V(s', w(s', x')) \mu(ds'|s) \\ \text{s.t.} \quad e &= p(s)n' + k' + c', \end{aligned} \tag{13}$$

where  $e$  is the *adjusted net worth* following the payout/default decision of the firm after the current state  $s$  has been observed (given past decision variables  $k$ ,  $n$ , and  $c$ ). The *realized net worth* can be written as

$$w(s, x) = \begin{cases} w_d(s) & \text{if } s \in \mathcal{S}_d \\ w_d(s) + [\pi(s) - \pi(s_d)] k^\alpha + [p(s) - p(s_d)] n & \text{if } s \in \mathcal{S}_d^c. \end{cases} \tag{14}$$

In (14),  $w_d(s)$  is the default threshold at  $s$ , to be defined later on. Given  $w_d(s)$ , we define the set

$$\mathcal{S}_d = \{s : w_d(s) \geq \pi(s)k^\alpha - \psi + p(s)n - h(n) + (1 - \delta)k + (1 + r)c\}$$

of the states in which the realized net worth is lower than the default threshold, and  $\mathcal{S}_d^c = \mathcal{S} \setminus \mathcal{S}_d$ . Because the default threshold is a contour in the  $(z, p)$ -space, there can be  $s_d^1 = (z_d^1, p_d^1)$  and  $s_d^2 = (z_d^2, p_d^2)$  such that  $s_d^1 \neq s_d^2$  and  $w(s_d^1, x) = w(s_d^2, x)$ . This is not a problem as far as the representation of  $w(s, x)$  in (14) is concerned, because, from  $w(s_d^1, x) = w(s_d^2, x)$ , we have  $\pi(s_d^1)k^\alpha + p(s_d^1)n = \pi(s_d^2)k^\alpha + p(s_d^2)n$ , and therefore the right-hand side of (14) in the case  $s \in \mathcal{S}_d^c$  is the same using either  $s_d^1$  or  $s_d^2$ .



The value function, which is the *cum-dividend* value of equity, is

$$V(s, w) = \max_e (1 + \lambda)(w - e)^- + (w - e)^+ + v(s, e), \quad (15)$$

where  $a^+ = \max\{a, 0\}$  and  $a^- = \min\{a, 0\}$ .  $V(s, \cdot)$  is a function of the *realized net worth*,  $w$ . From (15),  $e$  results from  $w$  by a dividend decision  $d = w - e$ , which takes into account the implications on the ensuing investment decisions through  $v(s, e)$ . However, if the optimal value on the right-hand side of (15) is negative, shareholders prefer to default and set their value,  $V(s, w)$ , at zero (limited liability). We define the default threshold  $w_d(s)$  on realized net worth by condition

$$V(s, w_d(s)) = 0. \quad (16)$$

This last condition closes the model.

First of all, it can be seen that the above description of the model is equivalent to that in (6)-(8). In particular, (14) is derived as follows. From

$$\begin{cases} 0 & \text{if } w \leq w_d(s) \\ V(s, w) & \text{if } w > w_d(s), \end{cases}$$

we can rewrite the realized net worth as

$$w(s, x) = \begin{cases} w_d(s) & \text{if } s \in \mathcal{S}_d \\ \pi(s)k^\alpha - \psi + p(s)n - h(n) + (1 - \delta)k + (1 + r)c & \text{if } s \in \mathcal{S}_d^c. \end{cases}$$

Using the definition of the default threshold in the  $(z, p)$ -space, we set

$$w_d(s) = \pi(s_d)k^\alpha - \psi + p(s_d)n - h(n) + (1 - \delta)k + (1 + r)c,$$

from which we can derive the second line in (14). To show that the optimal program is the same as in (6)-(8), it suffices to place  $v(s, e)$  from (13) in the right hand side of (15), and consider that the decision  $x' = (k', n', c')$  also determines the dividend,  $d = w - e = w - k' - pn' - c'$ .

**Proof of Proposition 1.** Following Cooley and Quadrini (2001), it can be seen that the solution of the program (13)-(16) exists and is unique. At a given  $s$ , we conjecture the existence of a lower bound  $\underline{w}(s)$  below which equity capital is raised and an upper bound  $\overline{w}(s)$  above which dividends are paid, with  $\underline{w}(s) < \overline{w}(s)$ . We will prove later on that this is indeed warranted. Based on this conjecture, we can restrict  $e \in [\underline{w}(s), \overline{w}(s)]$ .

Given decreasing returns to scale, there is an upper bound  $k_u$  such that  $k > k_u$  would not be economically profitable and would never be chosen in equilibrium. For similar reasons, given convex storage costs  $h(n)$ , there is an upper bound  $n_u$  such that  $n > n_u$  would never be chosen. Finally, because the return on cash holdings is lower than the return on equity ( $r < 1/\beta - 1$ ), also savings are bounded above by  $c_u$ . Because the domains of  $e$ ,  $k$ ,  $n$ ,  $c$ , and  $p$  are all bounded, the correspondence

$$\mathcal{F}(s, e) = \{(k', n', c') : k' \in [0, k_u], n' \in [0, n_u], c' \in [0, c_u], e = k' + pn' + c'\}$$

that defines the feasible set of the program in (13) is continuous, compact, and convex valued.

In problem (15), the payoff is continuous and strictly increasing in  $w$ . Thus,  $V(s, \cdot)$  is also strictly increasing in  $w$ , and we can properly define  $w_d(s)$  in (16). Using the same argument as in Proposition 5 of Hennessy and Whited (2007),  $w_d(s)$  is continuous and non-increasing.

From (13) we define the Bellman operator as

$$(Tv)(s, e) = \max_{x' \in \mathcal{F}(s, e)} \beta \int V(s', w(s', x')) \mu(ds'|s).$$

We now show that this operator maps the set of bounded and continuous functions into itself. Using the same argument as in Cooley and Quadrini (2001), this happens because, if  $v$  is continuous and bounded, then  $V$  is also continuous and bounded. The boundedness and continuity of  $\int w(s', x') \mu(ds'|s)$  and of  $V$  imply, together with the Feller property of  $\mu$ , that the objective function (13) is continuous and bounded. Because the correspondence  $\mathcal{F}$  is continuous, compact, and convex valued, the maximum exists and  $v$  is continuous (see Theorem 3.6 in Stokey and Lucas 1989). The resulting function  $Tv$  is unique because the operator  $T$  is a contraction. The proof of

this claim is straightforward showing that  $T$  satisfies Blackwell's sufficient conditions, following p. 1739 in Hennessy and Whited (2007). ■

**Lemma 1.** *The ex dividend value function  $v$  in (13) is strictly increasing, strictly concave, and differentiable with respect to  $e$ .*

**Proof of Lemma 1.** The argument follows the same logic as in Cooley and Quadrini (2001), so we refer the reader to their paper and report here the parts that are specific to our model. If  $v$  is concave and  $v(0) \geq 0$ , then  $V$  is strictly increasing and concave because the dividend  $(w - e)^+ + (w - e)^-(1 + \lambda)$  is strictly increasing and concave. As  $w$  is strictly increasing, then the compound function  $V \circ w$  is strictly increasing. Therefore,  $Tv$  is strictly increasing.

To show that  $v$  is strictly concave, Cooley and Quadrini (2001), on pp. 1306-1307, impose restrictions on the conditional distribution  $\mu(ds'|s)$ . Under these restrictions, to establish the strict concavity of  $V \circ w$  with respect to  $x = (k, n, c)$ , it is sufficient to show that  $\int w(s', x)\mu(ds'|s)$  is strictly concave with respect to  $x$ . Because we adopt the same distributional assumption on  $s'$  as in Cooley and Quadrini (2001) (in particular, we assume that the joint conditional distribution of  $(\log(z'), \log(p'))$  is Normal), the argument is also valid in our case. In particular, it can be seen that  $\int w(s', x)\mu(ds'|s)$  is strictly concave with respect to  $x$ . From a direct calculation, we have

$$\begin{aligned} \int w(s', x)\mu(ds'|s) &= \\ &= \int_{\mathcal{S}_d} w_d \mu(ds'|s) + \int_{\mathcal{S}_d^c} \{w_d + [\pi(s) - \pi(s_d)] k^\alpha + [p(s) - p(s_d)] n\} \mu(ds'|s) \\ &= (1 - \delta)k - \psi - h(n) + (1 + r)c + \mathbb{E}[\pi(s')|s] k^\alpha + \mathbb{E}[p(s')|s] n \\ &\quad + k^\alpha \int_{\mathcal{S}_d} [\pi(s_d) - \pi(s')] \mu(ds'|s) + n \int_{\mathcal{S}_d} [p(s_d) - p(s')] \mu(ds'|s). \end{aligned}$$

The first part,  $(1 - \delta)k - \psi - h(n) + (1 + r)c + \mathbb{E}[\pi(s')|s] k^\alpha + \mathbb{E}[p(s')|s] n$ , is strictly concave in  $(k, n, c)$ . The second part,  $\int_{\mathcal{S}_d} \{[\pi(s_d) - \pi(s')] k^\alpha + [p(s_d) - p(s')] n\} \mu(ds'|s)$ , under the distributional assumptions, is not very sensitive to changes in  $(k, n, c)$ , as in Lemma 1 in Cooley and Quadrini (2001). Therefore, the dominating part of  $\int w(z', x)\mu(ds'|s)$  is strictly concave, which is what we need.

Finally, the differentiability of  $v$  with respect to  $e$  is a consequence of Theorem 9.10 in Stokey and Lucas (1989). ■

**Proof of Proposition 2.** From Lemma 1,  $v$  is strictly concave and differentiable with respect to  $e$ . Therefore,  $\partial_e v(s, \cdot)$  is strictly decreasing. From the first-order conditions for the optimal  $e$  in (15), we can determine  $\underline{w}(s)$  from condition  $1 + \lambda = \partial_e v(s, \underline{w}(s))$  and  $\bar{w}(s)$  from  $1 = \partial_e v(s, \bar{w}(s))$ . Because  $\partial_e v(s, e) > 1 + \lambda$  for  $e < \underline{w}(s)$ , the optimal dividend in this case is  $w - \underline{w}(s) < 0$ , which takes the adjusted net worth at  $\underline{w}(s)$ . On the other hand, from  $\partial_e v(s, e) < 1$  for  $e > \bar{w}(s)$ , the optimal dividend in this case is  $w - \bar{w}(s) > 0$ , and the resulting adjusted net worth is  $\bar{w}(s)$ . ■

**Proof of Proposition 3.** Lemma 1 establishes strict monotonicity and concavity of  $v$ . Hence, the correspondence of the optimal policy is single-valued (i.e., for each  $(s, w)$  there is only one  $x' = (k', n', c')$  that maximizes (13)). From Proposition 2, the net worth is adjusted to stay within  $[\underline{w}(s), \bar{w}(s)]$ , so the optimal policy from program (13) coincides with the one at  $e = \underline{w}(s)$  for all  $w < \underline{w}(s)$ , and with the one at  $e = \bar{w}(s)$  for all  $w > \bar{w}(s)$ . ■

**Proof of Proposition 4.** We can establish the differentiability of  $V(s, \cdot)$  from the differentiability of  $v(s, \cdot)$ , see Lemma 1, and the fact that the payoff function of problem (15) is differentiable for values of  $e \neq w$ . When the dividend is zero,  $e = w$ , which occurs for  $w \in [\underline{w}(s), \bar{w}(s)]$ , we have  $V(s, w) = v(s, w)$ . Therefore, the differentiability of  $V(s, \cdot)$  in this case is a direct consequence of the differentiability of  $v(s, \cdot)$ . The function  $V$  is equal to  $v$  for  $w \in [\underline{w}(s), \bar{w}(s)]$ , so it is strictly concave in  $w$  in that region from Lemma 1, and is linear in  $w$  out of that region. ■

**Proof of Proposition 5.** To characterize the optimal policies, we introduce the Lagrangian function of the program (6)-(8) if the firm is currently solvent (i.e.  $w > w_d$ ):

$$\begin{aligned} \mathcal{L}(d, k', n', w') = & (1 + \lambda \chi_{\{d < 0\}}) d + \beta \int V(s', w') \mu(ds'|s) - \nu [d + p(s)n' + k' + c' - w] \\ & - \beta \int \nu(s') [w' - \pi(s')(k')^\alpha + \psi - p(s')n' + h(n') - (1 - \delta)k' - (1 + r)c'] \mu(ds'|s), \end{aligned}$$

where  $\nu$  is the Lagrange multiplier of constraint (8), and  $\beta \mu(s'|s) \nu(s')$  is the multiplier of (7). The first-order conditions with respect to the decision variables are

$$\nu = 1 + \lambda \chi_{\{d < 0\}},$$

$$\nu(s') = \partial_w V(s', w') \quad \text{for all } s' \in \mathcal{S},$$

$$\beta \int \nu(s') [\pi(s') \alpha(k')^{\alpha-1} + (1 - \delta)] \mu(ds'|s) - \nu \leq 0, \quad (17)$$

$$\beta \int \nu(s') [p(s') - h'(n')] \mu(ds'|s) - \nu p(s) \leq 0, \quad (18)$$

$$\beta(1 + r) \int \nu(s') \mu(ds'|s) - \nu \leq 0, \quad (19)$$

together with the complementary slackness conditions

$$k' \left\{ \beta \int \nu(s') [\pi(s') \alpha(k')^{\alpha-1} + (1 - \delta)] \mu(ds'|s) - \nu \right\} = 0, \quad (20)$$

$$n' \left\{ \beta \int \nu(s') [p(s') - h'(n')] \mu(ds'|s) - \nu p(s) \right\} = 0, \quad (21)$$

$$c' \left\{ \beta(1 + r) \int \nu(s') \mu(ds'|s) - \nu \right\} = 0. \quad (22)$$

Because the production function,  $\pi(s')(k')^\alpha$ , satisfies the Inada conditions, then  $k' > 0$ . Therefore, both  $\underline{k}(s)$  and  $\bar{k}(s)$  in Proposition 3 are strictly positive. As  $k' > 0$ , from (20) and (17), then

$$\beta \int \nu(s') [\pi(s') \alpha(k')^{\alpha-1} + (1 - \delta)] \mu(ds'|s) = \nu.$$

From (18), if the expected marginal value of inventory is strictly lower than  $\nu$ ,

$$\beta \int \nu(s') \left[ \frac{p(s') - h'(n')}{p(s)} \right] \mu(ds'|s) < \nu,$$

then, using condition (21) we can conclude that  $n' = 0$ . Otherwise, if  $n' > 0$ , then

$$\beta \int \nu(s') \left[ \frac{p(s') - h'(n')}{p(s)} \right] \mu(ds'|s) = \nu. \quad (23)$$

Proposition 5 follows on immediately from these.

We can characterize the optimal saving policy at state  $(s, w)$ . For this purpose, we define  $\Phi(s) = (1 + r)\beta \int \nu(s') \mu(ds'|s)$  the expected marginal benefit of  $c'$  (i.e., the present value of  $1 + r$  dollar in  $t + 1$ , given the different possible financing states) conditional on the current state  $s$ . Notably,

$\Phi(s)$  is only a function of  $s$ , not of  $w$ , and from Proposition 2,  $\beta(1+r) \leq \Phi(s) \leq \beta(1+r)(1+\lambda)$ .

From (19), we have

$$\Phi(s) \leq \nu = \begin{cases} 1 & \text{if } d \geq 0 \\ 1 + \lambda & \text{if } d < 0 \end{cases}.$$

Panel A of Figure 7 describes, for a given  $(s, w)$ , the optimal saving policy,  $c'$ , assuming the optimal policy for  $k'$  and  $n'$  as given. We compare  $\Phi(s)$  to the marginal cost,  $\nu$ . The marginal cost increases from 1 to  $1 + \lambda$  when  $c'$  increases, because the firm goes from a state in which it pays dividends to a state in which it issues equity. In between these extremes, there are states in which the dividend is zero. In these states, using equation (8), we define  $c'_0 = w - p(s)n' - k'$  the cash surplus available from  $w$ , given  $k'$  and  $n'$  have been already decided. In the figure, the marginal cost is represented by the two dashed horizontal lines. The downward sloping solid curve is the expected marginal benefit, which is decreasing because more savings reduce the probability of hitting equity issuance states. Moreover, for very large  $c'$ , the expected benefit becomes  $\beta(1+r)$ , which is lower than 1, given our assumption on the return on savings. The point at which this line intersects the cost line of level 1 is  $\bar{c}(s)$ . The point at which the benefit line intersects the cost line of level  $1 + \lambda$  is  $\underline{c}(s) = 0$ .

The optimal cash policy is found by equating cost and benefit of  $c'$ , and is determined by  $c'_0$ , for which there are three possible scenarios. If the cash surplus is negative,  $c'_0 < 0$ , because the benefit is higher than the cost, the manager raises  $-c'_0$  by issuing equity, and the optimal savings is  $c' = 0$ . With this decision, the firm achieves the adjusted net worth  $\underline{w}(s)$ . In the second scenario,  $c'_0 > \bar{c}(s)$ , given the significant cash surplus after all the other decisions have been made. Because the marginal cost is higher than the marginal benefit at this point, then  $c' = \bar{c}(s)$ ,  $c'_0 - \bar{c}(s)$  is paid as dividend, and the adjusted net worth becomes  $\bar{w}(s)$ . Finally, if  $c'_0 \in [0, \bar{c}(s)]$ , then  $c' = c'_0$  and the adjusted net worth is  $w$ . Notably, because depending on  $s$   $\Phi(s)$  can vary in between  $\beta(1+r)$  and  $\beta(1+r)(1+\lambda)$ , the marginal benefit curve can have different slopes, represented by the two dashed lines  $\Phi(s_1)$  and  $\Phi(s_2)$ .

As for the optimal inventory policy at  $(s, w)$ , in Panel B of Figure 7 we denote  $\Psi(s)$  the marginal benefit of inventory, which is the left-hand side of equation (23). As before, we consider the marginal effect of changing  $n'$ , assuming the optimal decisions on  $k'$  and  $c'$  have been already made. In the

figure, the benefit is again a decreasing function for the reasons described above. The cost, going from low to high  $n'$ , increases from 1 to  $1 + \lambda$ , because the current dividend goes from positive to negative. When the dividend is zero, we define  $n'_0$  from equation (8) as  $n'_0 = (w - k' - c')/p(s)$  as the inventory that can be purchased from  $w$  given the optimal  $k'$  and  $c'$ .

In the figure, the benefit curve depends on the current state because for a high price  $p(s)$  the benefit is reduced. We first consider a state  $s_1$  in which the price  $p(s)$  is relatively low, so that the benefit curve  $\Psi(s_1)$ . There are three scenarios for  $n'_0$ . If it is lower than  $\underline{n}(s_1)$ , because the marginal benefit is higher than the marginal cost the manager issues equity to finance the purchase of  $n' = \underline{n}(s_1)$  and the adjusted net worth becomes  $\underline{w}(s)$ . In the second scenario, for  $n'_0 > \bar{n}(s_1)$ , the firm chooses  $n' = \bar{n}(s_1)$  by paying out the excess  $w$ , because the (opportunity) cost is higher than the benefit, so that the adjusted net worth is  $\bar{w}(s)$ . Finally, if  $n'_0 \in [\underline{n}(s_1), \bar{n}(s_1)]$ ,  $n' = n'_0$  and no dividend is paid.

In a scenario with relatively higher commodity price, like  $s_2$ , the marginal benefit of inventory is lower and the curve is shifted to the left so that it intersects the curve of level at a negative  $n'$ , as represented in in Panel B of Figure 7. In this case, differently from the previous case, if  $n'_0 = 0$ , then optimal inventory  $n'$  is zero, although for a higher  $w$  inventory will be strictly positive. A third and final scenario (not represented in Figure 7) is the one with high  $p(s)$ , in which both intersections of the benefit curve with the cost function are in the negative quadrant. In this scenario, the benefit of inventory is so low (in the current state) that there is no level of net worth for which the firm invests in inventory. ■

## B. Numerical methods

Given the properties of the value function, we solve (6)-(8) using a successive approximations method to find  $V$  and the optimal policies for capital, inventory, and cash holdings. We discretize the capital set in 61 points chosen as  $k_u(1 - \delta)^j/2$ , for  $j = 1, \dots, 61$ . The sets of inventory and cash holdings are discretized in  $[0, n_u]$  and  $[0, c_u]$  with 61 equally spaced points. The exogenous variables  $z$  and  $p$  define a reduced-form vector autoregression that we approximate through a discrete-state Markov chain with 9 points for each variable with truncated support in  $[-3\sigma_j^u, 3\sigma_j^u]$ ,  $j = p, z$ , where  $\sigma_j^u = \sigma_j/\sqrt{1 - \phi_j^2}$  is the unconditional standard deviation for  $j = p, z$ . The discrete abscissae and

the risk-neutral Markov transition probabilities are computed according to the method proposed by Terry and Knotek (2011), which is based on the Gauss-Hermite quadrature rule, as in Tauchen (1986), but allows for non-zero correlation.

A Monte Carlo simulation is used to generate a sample of 50 simulated economies, each comprising 10,000 firms observed at the steady state over 150 periods. To generate the history of one firm in one economy, we initialize a path at a random  $(z_0, p_0)$  and at an initial common state,  $(k_0, n_0, c_0)$ . At each  $t = 0, 1, 2, \dots$ , given  $(z_t, p_t, k_t, n_t, c_t)$ , we apply the optimal policy from the optimal program in (6) and find  $(k_{t+1}, n_{t+1}, c_{t+1})$ . Next, we draw from the truncated bi-variate Normal distribution described above and generate the next value of productivity and price along the path,  $(z_{t+1}, p_{t+1})$ , using the AR(1) specification. We repeat this 250 times. For each firm's history, we drop the first 100 observations, to exclude any influence of the initial condition,  $(k_0, n_0, c_0)$ , and retain the last 150 observations. To keep the number of firms in an economy constant, in the event of default, a new company enters the market in place of the old one. The new company is endowed with a level of capital equal to the intermediate value of the grid of  $k$ , with no inventory,  $n = 0$ , and with no cash holdings,  $c = 0$ . This choice allows the new firm to be considered as a relatively small unhedged company.

### C. Inventory and derivatives

Inventory and derivatives are quite different, as far as the frictions they entail are considered. While inventory requires an investment in the current period and generates storage costs, derivatives are subject to basis risk and limits imposed by the firm's counterparty risk, such as collateral constraints, margin requirements, or premia on the price paid by the firm.

Still, inventory and derivatives are similar in that they are both contingent hedging tools. With inventory, in state  $s$  the manager invests an amount  $p(s)n'$  and will obtain a payoff  $p(s')n' - h(n')$  at the end of the period. Using, for instance, a forward contract, the manager does not pay anything at the inception and will receive a payoff  $m'(p(s') - p_f)$  in the next period, where  $m'$  is the amount invested in the contract and  $p_f$  is the forward price agreed in the current period. For the moment, to allow for this comparison we do not consider a collateral constraint. This implies the counterparty is exposed to the credit risk of the firm, and would charge a premium (i.e., a relative higher  $p_f$ ).



The optimality conditions for inventory in (10) can be rewritten

$$\beta \int \partial_w V(s', w') p(s') \mu(ds'|s) = (1 + \lambda \chi_{\{d < 0\}}) p(s) + h'(n') \Lambda(s),$$

where for convenience we define  $\Lambda(s) = \beta \int \partial_w V(s', w') \mu(ds'|s)$ , the present value, conditional on  $s$ , of the opportunity cost of capital at the end of the period. The same condition in case the manager used forward contracts in place of inventory would be

$$\beta \int \partial_w V(s', w') p(s') \mu(ds'|s) = p_f \Lambda(s).$$

As the expected benefits of the two hedges are the same (the left-hand sides of the equations above), we can focus on the right-hand sides to compare their relative efficiency. Inventory requires a payment at the current date, which is valued  $(1 + \lambda \chi_{\{d < 0\}}) p(s)$  by the shareholders, and another payment at the end of the period, with value  $h'(n') \Lambda(s)$ . In contrast, the forward affects only the net worth at the end of the period, with a value today of  $p_f \Lambda(s)$ . This means that if derivatives were included in our model, their role relative to inventory would be determined mostly by the frictions they generate (among these, the most important are storage costs and credit risk), and our analysis would remain the same.

However, if we added a collateral constraint, which would occur if we considered futures in place of forwards, things would be different. In this case, as proved by Rampini, Sufi, and Viswanathan (2014), if current net worth is sufficiently low, it is optimal for the manager *not* to hedge some future states, even if they are insurable. In contrast, because inventory does not require any collateral, Proposition 5 states that if the expected marginal value of inventory is high enough, which depends on the commodity price and *not* on net worth, inventory investment is strictly positive for all  $e > \underline{w}(s)$ . Hence, absence of risk management is related to the cost of the hedging instrument, rather than the adjusted net worth of the firm.

To conclude, including hedging using derivatives in our model would not rule out inventory and therefore would not change our qualitative findings. Low net worth firms would abstain from using derivatives (either because of credit risk charges or collateral constraints), and therefore would favor the use of inventory. Firms with higher net worth may also prefer inventory to derivatives if

basis risk reduces the effectiveness of financial hedges. Hence, inventory would be used even when the integrated risk management policy involved derivatives.

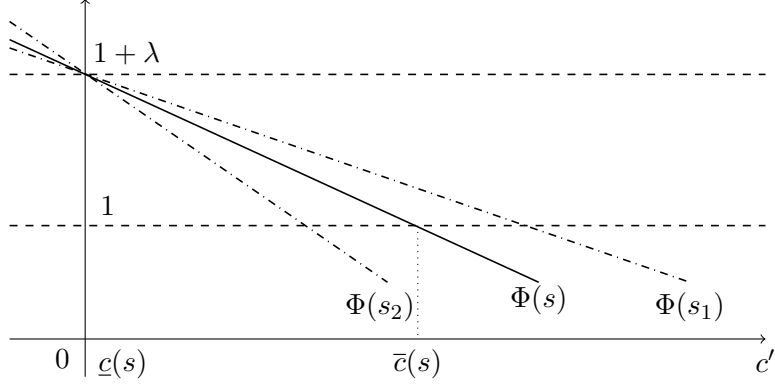
#### D. Shock persistence and investment

We investigate the role of persistence (of productivity and price) in risk management using inventory. In Figure 8, we show investment in capital and inventory against  $w$  for different levels of persistence of productivity,  $\phi_z$ , and of commodity price,  $\phi_p$ , in different current states. In Panels A and B, for the comparative static on  $\phi_z$ , we set  $p$  relatively low (so there is an incentive to invest in inventory in some states), and choose  $z$  either below or above its unconditional mean. We observe that, for high  $z$ , the manager invests more in  $k'$  and less in  $n'$ . This is because, being  $z$  persistent, a high current state predicts high future cash flows, giving an incentive to invest in capital stock. At the same time, the manager invests less in inventory because high capital and high  $z'$  will make internal financing of future investments more likely, and because the current opportunity cost of risk management is high. This reasoning (with opposite sign) also explains the policies when  $z$  is low. Notably, the role of commodity in production would give the opposite prediction of high investment in commodity when total factor productivity is high.

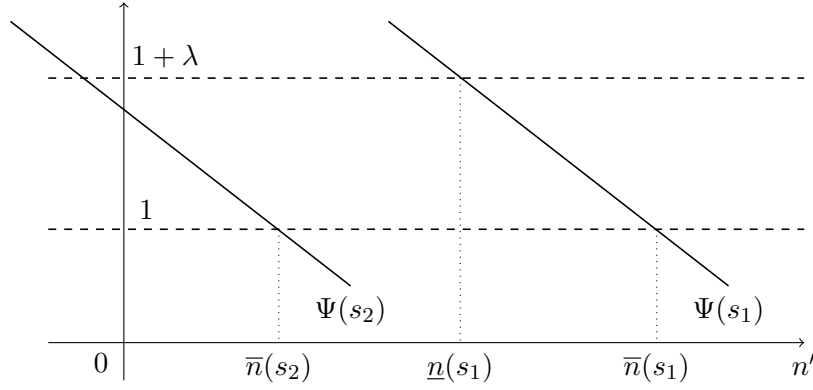
When we compare different levels of  $\phi_z$ , investment is higher and there is a lower incentive to manage risk if persistence is higher and  $z$  is high (and vice versa if  $z$  is low), because it is more likely that future cash flows will be high. This increases the present value of investment and makes the opportunity cost of risk management higher.

In Panels C and D for the comparative static on  $\phi_p$ , we set  $z$  at its unconditional mean and choose  $p$  either below or above its unconditional mean. For low  $p$ , investment in  $k'$  and investment in  $n'$  are both high. Given the persistence of  $p$  and  $z$ , high current actual productivity,  $\pi$ , predicts high future actual productivity, making the NPV of investment positive. While this reduces the future marginal value of net worth, and therefore the demand for risk management, a low  $p$  also increases the marginal value of inventory (see Proposition 5), thus increasing the investment in storage. The two opposing incentives are both present in our model, and in this case the latter prevails.

Comparing different levels of  $\phi_p$ , if  $p$  is low, higher persistence increases the incentive to invest, because high future cash flows become more likely. For exactly the same reason, risk management is less needed if  $p$  is more persistent. Again, the behavior in Panel D can only be explained by the fact that inventory is used for risk management purposes.



(A) Cash holding policy



(B) Inventory policy

Figure 7: **Optimal saving and inventory policies.** In Panel A we plot the marginal cost and benefit of  $c'$  at  $(s, w)$  and for given  $k'$  and  $n'$ . In Panel B we plot the marginal cost and benefit of  $n'$  at  $(s, w)$  and for given  $k'$  and  $c'$ . In both panels,  $\nu$  is either 1 or  $1 + \lambda$ . In Panel A,  $\Phi(s) = (1 + r)\beta \int \nu(s')\mu(ds'|s)$  is the expected marginal benefit of cash holdings. In Panel B,  $\Psi(s) = \beta \int \nu(s')[p(s') - h'(n')]/p(s)\mu(ds'|s)$  is the expected marginal benefit of inventory.

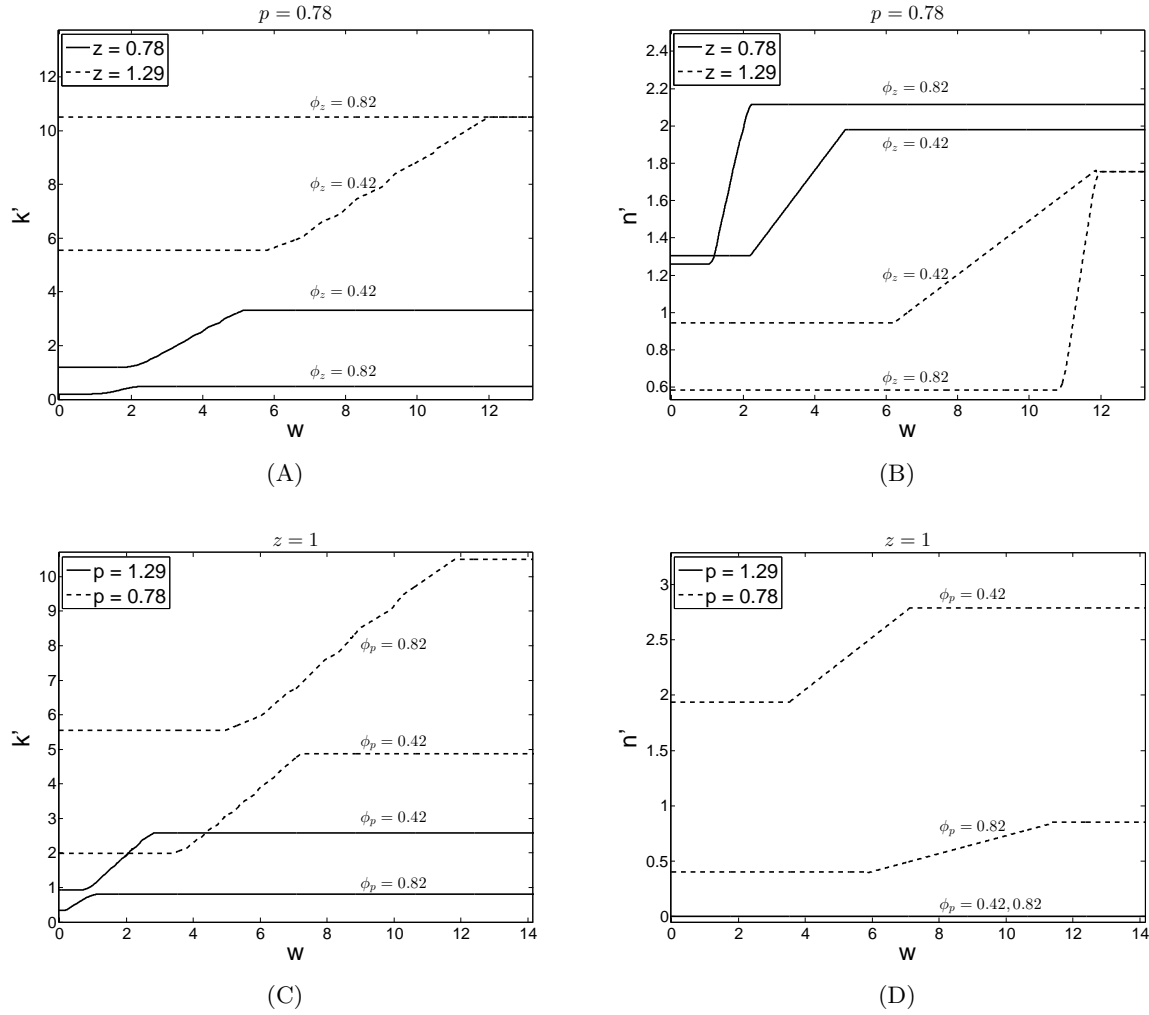


Figure 8: **The effect of persistence.** We plot against net worth  $w$  the optimal  $k'$  and  $n'$  for different values of  $\phi_z$  (Panels A and B) and  $\phi_p$  (Panels C and D), at different current states. In Panels A and B,  $p = 0.78$  and the productivity shock can either be  $z = 0.78$  or  $z = 1.29$ . In Panels C and D,  $z = 1$ , and the price can either be  $p = 0.78$  or  $p = 1.29$ . All the other parameters are as in Table I.